

u-Substitution - Homework

1. $\int \sqrt{x-2} \, dx$

$$\int u^{1/2} du \quad u = x-2, \, du = dx$$

$$\frac{2}{3} u^{3/2} = \frac{2(x-2)^{3/2}}{3} + C$$

2. $\int (2x+3)^{11} \, dx$

$$\frac{1}{2} \int u^{11} du \quad u = 2x+3, \, du = 2dx$$

$$\frac{1}{2} \left(\frac{u^{12}}{12} \right) = \frac{(2x+3)^{12}}{24} + C$$

3. $\int \sqrt{5x-1} \, dx$

$$\frac{1}{5} \int u^{1/2} du \quad u = 5x-1, \, du = 5dx$$

$$\left(\frac{1}{5} \right) \left(\frac{2}{3} \right) u^{3/2} = \frac{2(5x-1)^{3/2}}{15} + C$$

4. $\int \sqrt[3]{6x+1} \, dx$

$$\frac{1}{6} \int u^{1/3} du \quad u = 6x+1, \, du = 6dx$$

$$\frac{1}{6} \left(\frac{3}{4} \right) u^{4/3} = \frac{(6x+1)^{4/3}}{8} + C$$

5. $\int 5(3-4x)^{2/3} \, dx$

$$\frac{-1}{4} \int 5u^{2/3} du \quad u = 3-4x, \, du = -4dx$$

$$\frac{-1}{4} (5) \left(\frac{3}{5} \right) u^{5/3} = \frac{-3(3-4x)^{5/3}}{4} + C$$

6. $\int \frac{dx}{(8x-1)^3}$

$$\frac{1}{8} \int u^{-3} du \quad u = 8x-1, \, du = 8dx$$

$$\frac{1}{8} \left(\frac{u^{-2}}{-2} \right) = \frac{-1}{16(8x-1)^2} + C$$

7. $\int x(x^2+2)^6 \, dx$

$$\frac{1}{2} \int u^6 du \quad u = x^2+2, \, du = 2x \, dx$$

$$\frac{1}{2} \left(\frac{u^7}{7} \right) = \frac{(x^2+2)^7}{14} + C$$

8. $\int 6x^2 \sqrt{3x^3-1} \, dx$

$$\frac{1}{9} \int 6u^{1/2} du \quad u = 3x^3-1, \, du = 9x^2 \, dx$$

$$6 \left(\frac{1}{9} \right) \left(\frac{2}{3} \right) u^{3/2} = \frac{4(3x^3-1)^{3/2}}{9} + C$$

9. $\int \left(1 + \frac{1}{x} \right)^3 \left(\frac{1}{x^2} \right) dx$

$$-\int u^3 du \quad u = 1 + \frac{1}{x}, \, du = \frac{-1}{x^2} \, dx$$

$$-\left(\frac{u^4}{4} \right) = \frac{-\left(1 + \frac{1}{x} \right)^4}{4} + C$$

10. $\int x^{1/3} (x^{4/3} + 9)^8 \, dx$

$$\frac{3}{4} \int u^8 du \quad u = x^{4/3} + 9, \, du = \frac{4}{3} x^{1/3} \, dx$$

$$\frac{3}{4} \left(\frac{u^9}{9} \right) = \frac{(x^{4/3} + 9)^9}{12} + C$$

11. $\frac{2}{3} \int \sqrt{4 - \frac{3}{5}x} \, dx$

$$\frac{2}{3} \left(\frac{-5}{3} \right) \int u^{1/2} du \quad u = 4 - \frac{3}{5}x, \, du = \frac{-3}{5} \, dx$$

$$\frac{2}{3} \left(\frac{-5}{3} \right) \left(\frac{2}{3} \right) u^{3/2} = \frac{-20 \left(4 - \frac{3}{5}x \right)^{3/2}}{27} + C$$

12. $\int (3x+15)\sqrt{x^2+10x+4} \, dx$

$$\frac{3}{2} \int (x+5)u^{1/2} du \quad u = x^2+10x+4, \, du = 2(x+5) \, dx$$

$$\frac{3}{2} \left(\frac{2}{3} \right) u^{3/2} = (x^2+10x+4)^{3/2} + C$$

$$13. \int (x+2)\sqrt{x-2} dx$$

$$\int (u+2+2)u^{1/2} du \quad u = x-2, \quad du = dx \quad x = u+2$$

$$\int (u^{3/2} + 4u^{1/2}) du$$

$$\frac{2}{5}u^{5/2} + 4\left(\frac{2}{3}\right)u^{3/2} = \frac{2(x-2)^{5/2}}{5} + \frac{8(x-2)^{3/2}}{3} + C$$

$$14. \int \frac{x^2}{\sqrt{x-4}} dx$$

$$\int (u+4)^2 u^{-1/2} du \quad u = x-4, \quad du = dx \quad x = u+4$$

$$\int (u^2 + 8u + 16)u^{-1/2} du$$

$$\int (u^{3/2} + 8u^{1/2} + 16u^{-1/2}) du$$

$$\frac{2}{5}u^{5/2} + 8\left(\frac{2}{3}\right)u^{3/2} + 16(2)u^{1/2}$$

$$\frac{2(x-4)^{5/2}}{5} + \frac{16(x-4)^{3/2}}{3} + 32(x-4)^{1/2} + C$$

$$15. \int \sin 5x dx$$

$$\frac{1}{5} \int \sin u du \quad u = 5x, \quad du = 5 dx$$

$$\frac{1}{5}(-\cos u) = \frac{-\cos 5x}{5} + C$$

$$16. \int \cos \frac{x}{2} dx$$

$$2 \int \cos u du \quad u = \frac{x}{2}, \quad du = \frac{1}{2} dx$$

$$2(\sin u) = 2 \sin \frac{x}{2} + C$$

$$17. \int \frac{1}{3} \sec^2 8x dx$$

$$\frac{1}{3} \left(\frac{1}{8} \right) \int \sec^2 u du \quad u = 8x, \quad du = 8 dx$$

$$\frac{1}{3} \left(\frac{1}{8} \right) (\tan u) = \frac{\tan 8x}{24} + C$$

$$18. \int \sin 4x \cos 4x dx$$

$$\frac{1}{4} \int \sin u \cos u du \quad u = \sin 4x, \quad du = 4 \cos 4x dx$$

$$\frac{1}{4} \int u du = \frac{1}{4} \left(\frac{u^2}{2} \right) = \frac{\sin^2 4x}{8} + C \quad \text{or} \quad \frac{-\cos^2 4x}{8} + C$$

$$19. \int \cos^3 x \sin x dx$$

$$-\int u^3 du \quad u = \cos x, \quad du = -\sin x dx$$

$$-\left(\frac{u^4}{4}\right) = \frac{-\cos^4 x}{4} + C$$

$$20. \int \tan x \sec^2 x dx$$

$$\int u du \quad u = \tan x, \quad du = \sec^2 x dx$$

$$\frac{u^2}{2} = \frac{\tan^2 x}{2} + C$$

$$21. \int \sqrt{\cos 6x} \sin 6x dx$$

$$-\frac{1}{6} \int u^{1/2} du \quad u = \cos 6x, \quad du = -6 \sin x dx$$

$$-\frac{1}{6} \left(\frac{2}{3} \right) (u^{3/2}) = \frac{-(\cos(6x))^{3/2}}{9} + C$$

$$22. \int \frac{\sin x}{(4 - \cos x)^3} dx$$

$$\int u^{-3} du \quad u = 4 - \cos x, \quad du = \sin x dx$$

$$\frac{u^{-2}}{-2} = \frac{-1}{2(4 - \cos x)^2} + C$$