2008 AP Calculus AB Multiple Choice Exam

Section 1

No Calculator Active

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Section 2

Calculator Active

1. $\lim_{x\to\infty}\dfrac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

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2. $\int \dfrac{1}{x^2}\,dx =$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

3. If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

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4. $\int(\sin(2x)+\cos(2x))\,dx=$

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(B) $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$

(E) $-2\cos(2x)+2\sin(2x)+C$

5. $\lim_{x\to 0}\dfrac{5x^4+8x^2}{3x^4-16x^2}$ is

(A) $-\dfrac{1}{2}$ (B) 0 (C) 1 (D) $\dfrac{5}{3}+1$ (E) nonexistent

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$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$.

II. f is continuous at $x=2$.

III. f is differentiable at $x=2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

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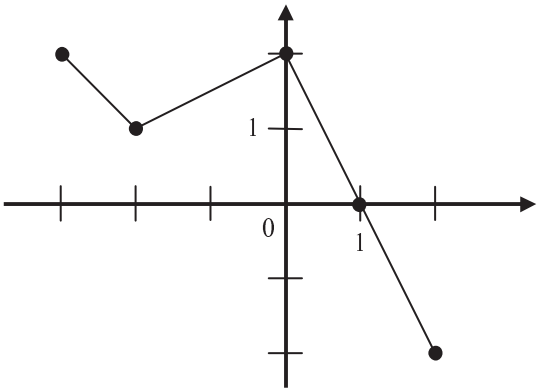
8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

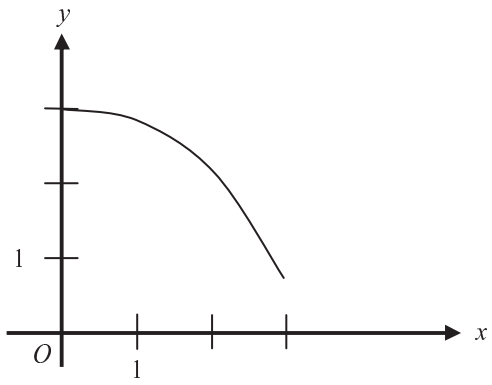
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$



Graph of f

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

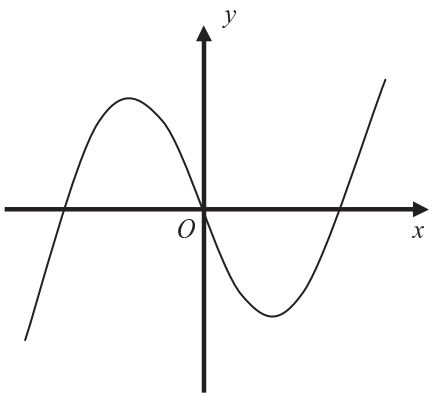
(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

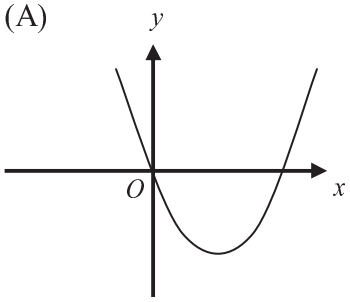
(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length



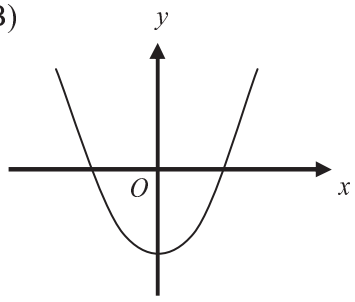
Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

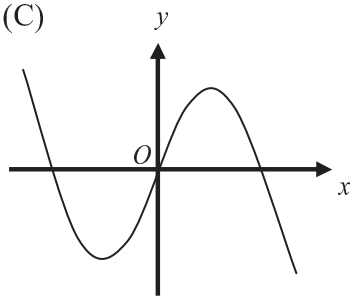
(A)



(B)



(C)



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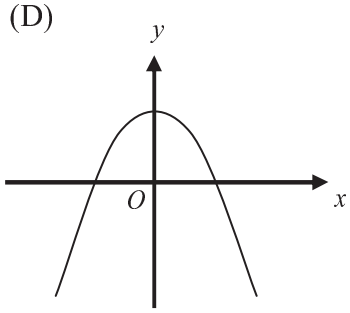
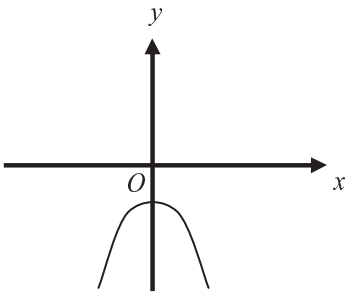


(E)



12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2} e^{(2/x)}$ (E) $-2x^2 e^{(2/x)}$

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13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}\left(f(\ln x)\right) =$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x \ln x + 2$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x + 2}{x}$

x	0	1	2	3
$f''(x)$	5	0	−7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

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15. $\int \frac{x}{x^2 - 4}\,dx =$

(A) $\frac{-1}{4(x^2 - 4)^2} + C$

(B) $\frac{1}{2(x^2 - 4)} + C$

(C) $\frac{1}{2}\ln|x^2 - 4| + C$

(D) $2\ln|x^2 - 4| + C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

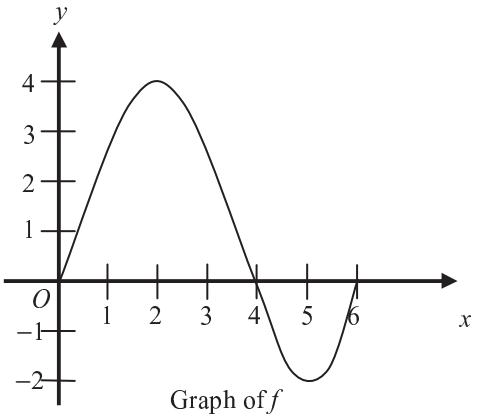
(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

18. In the xy-plane, the line $x+y=k$, where k is a constant, is tangent to the graph of $y=x^2+3x+1$. What is the value of k ?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

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19. What are all horizontal asymptotes of the graph of $y=\frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y=-1$ only

(B) $y=0$ only

(C) $y=5$ only

(D) $y=-1$ and $y=0$

(E) $y=-1$ and $y=5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

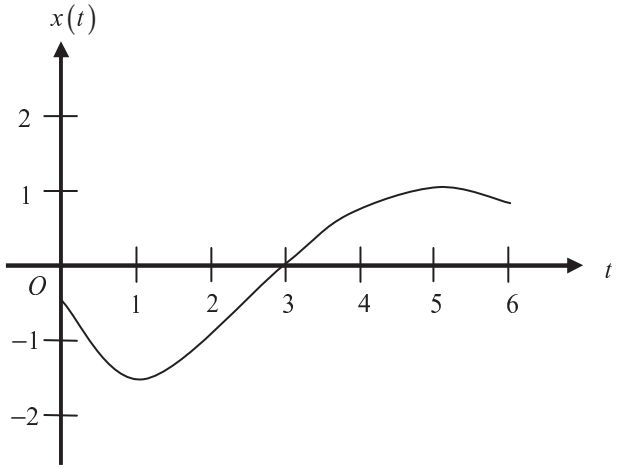
(A) 0 only

(B) 3 only

(C) 0 and 6 only

(D) 3 and 6 only

(E) 0, 3, and 6



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N - p)$

(C) $\frac{dp}{dt} = kp(p - N)$

(D) $\frac{dp}{dt} = kt(N - t)$

(E) $\frac{dp}{dt} = kt(t - N)$

23. Which of the following is the solution to the differential equation $\frac{dy}{dx}=\frac{x^2}{y}$ with the initial condition $y(3)=-2$?

(A) $y=2e^{-9+x^3/3}$

(B) $y=-2e^{-9+x^3/3}$

(C) $y=\sqrt{\frac{2x^3}{3}}$

(D) $y=\sqrt{\frac{2x^3}{3}-14}$

(E) $y=-\sqrt{\frac{2x^3}{3}-14}$

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24. The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$$f(x)=\begin{cases} cx+d & \text{for } x\le 2 \\ x^2-cx & \text{for } x>2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x=2$, what is the value of $c+d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

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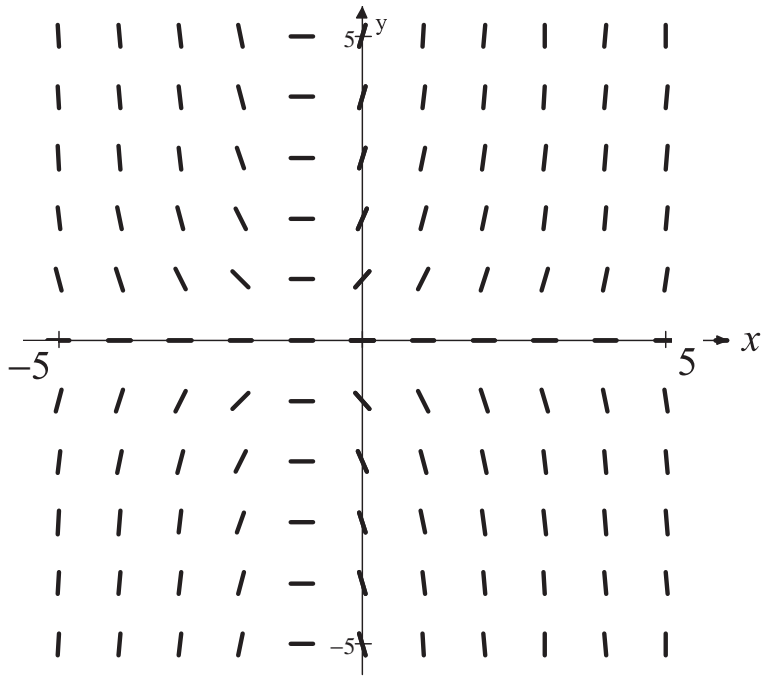
26. What is the slope of the line tangent to the curve $y=\arctan(4x)$ at the point at which $x=\frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2



27. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$

28. Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.
