Name____ Class__________ Date________________

AP Calculus ~ Semester I Review
Part 1

All problems are to be worked without a calculator unless otherwise noted.

Limits

Determine the following limits by direct evaluation, factoring and reducing, L'Hoptital's Rule or the definition of a derivative.

1. $\lim_{x\to 1}\frac{x^3-1}{x-1}$

2. $\lim_{x\to -3}\frac{x^2+x-6}{x+3}$

3. $\lim_{x\to \infty}\frac{\sqrt{4x^2+1}}{3x+2}$

__________ __________ __________

4. $\lim_{x\to -2}\frac{x^3-2}{x-1}$

5. $\lim_{x\to 0}\frac{\sin x}{x}=$

6. $\lim_{x\to 0}\frac{1-\cos x}{x}=$

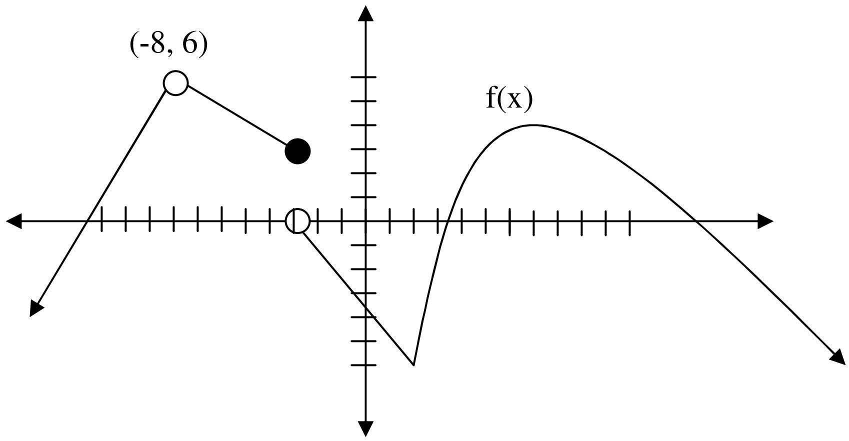
__________ __________ __________

7. $\lim_{\Delta x\to 0}\frac{\left[(x+\Delta x)^3+2(x+\Delta x)\right]-\left[(x)^3+2(x)\right]}{\Delta x}$

8. $\lim_{x\to 0}\frac{\sin 4x}{x}=$

9. $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$

__________ __________ __________



Answer questions 10-14 from the above graph of f(x).

10. $\lim_{x \to -8} f(x)$

11. $\lim_{x \to -3} f(x)$

12. $\lim_{x \to -3^-} f(x)$

13. Name the continuous intervals for f(x).

14. Name the differentiable intervals for f(x).

Derivatives

15. Find f'(2) for $f(x) = (x^2 - 2x + 1)(x^3 - 1)$.

16. Find g'(2) for $g(x) = \dfrac{3 - 2x - x^2}{x^2 - 1}$.

17. Find h'(3) for $h(x) = (3x - 2x^2)^3$.

18. Find q'(x) for $q(x) = x^2\sqrt{1 - x^2}$

Complete the following identities.

19. $\frac{d}{dx}\tan x =$ _______ 20. $\frac{d}{dx}\sec x =$ ____________ 21. $\frac{d}{dx}\cot x =$ _______

22. $\frac{d}{dx}\csc x =$ ____________ 23. $\frac{d}{dx}e^{u} =$ _______

24. $\frac{d}{dx}\cos^2(x^3) =$ ____________ 25. If $g(x) = \tan e^{-x}$, $g'(x) =$_________

26. If $f(x) = \tan(3x)$, find $f'(\frac{\pi}{9}) =$_________ 27. If $f(x) = \cos e^{2x}$, find $f'(x) =$_________

28. State continuous and differentiable intervals for $f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2 - 3x, & x \geq 1 \end{cases}$.

29. If $f(x) = \begin{cases} x + k, & x \leq 2 \\ x^2 + 1, & x > 2 \end{cases}$, find k so that f is continuous.

30. If $f(x) = \begin{cases} ax - 2, & x \leq 3 \\ x^2 - b, & x > 3 \end{cases}$, find a and b so that f(x) is continuous and differentiable.

31. Given that f and g are differentiable functions (-∞, ∞), g(x) <0 for all x's, and f(0)=2,
if $h(x) = \frac{f(x)}{g(x)}$ and $h'(x) = \frac{-f(x)g'(x)}{[g(x)]^2}$ what can you conclude about f?

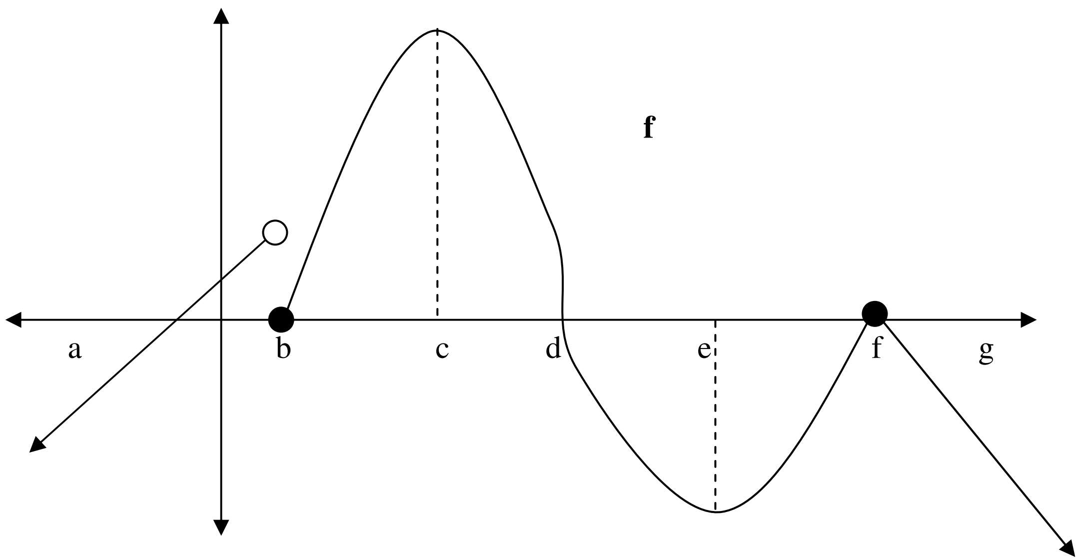
____

32. Find the instantaneous rate of change at t = 3 for $h(t) = \dfrac{t^3 - 3}{t - 2}$.

(A calculator is allowed on this problem.)

____

33. The graph of f has horizontal tangents at c and e, and a vertical tangent at d as indicated on the graph below.



a) On what intervals (-∞, ∞) is f continuous?

b) On what intervals (a, g) is f differentiable?

c) On what intervals is f ‘(x) < 0?

For questions 34-35 use the following data: $f(2) = 3,\ f\,'(2) = -1,$ g(2)=5, g(1)=2, g'(2)=8, $g\,'(1) = 4$**.**

34. Find h ‘(2) if h(x) = f(x) g(x).

35. Find t ‘(1) if t(x)= f(g(x))

36. Find a local linear approximation for $f(x) = \tan x$ at $x = \pi$ and use it to approximate $f(3)$.

37. Find a local linear approximation for $g(x) = \ln (3x)$ at $x = e$ and use it to approximate $g(2)$.

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AP Calculus ~ Semester I Review
Part 2
Implicit Derivatives and Related Rates

1. Find the slope of a line tangent to the curve $x^2 y + xy = 8$ at the point (1, 4).

2. Find d^2y/dx^2 for $x^2 + xy = 5$

Related Rates

(A calculator is allowed on problems 3-7.)

3. A spherical balloon is being inflated at a rate of 20 cubic feet per minute.

a) In terms of circumference what is the rate at which the radius is changing in ft/min ?

b) How fast is the radius increasing at the instant the radius is 2 ft?

4. A 10 foot ladder is leaning against a vertical wall. If the foot of the ladder slides away from the base of the wall at a rate of 1 $^{ft}/_{s}$, how fast is the top of the ladder sliding down the wall when the ladder is 6ft from the wall?

At what rate is the angle between the top of the ladder and the wall changing at this time?

5. A forklift with a height of 5 ft is moving toward a light that is 20 foot off the floor, at a rate of 2 feet per second. At what rate is the shadow of the forklift shrinking at the moment the forklift is 15 feet from the point on the floor directly under the light?

At what rate is the farthest tip of its shadow approaching the point on the floor directly under the light at this time?

6. A minimum wage employee is filling a conical cup with pineapple flavored ice at a rate of 10 cubic centimeters per second. The conical cup has a diameter of 12 centimeters and a depth of 16 centimeters. At what rate is the depth of flavored ice in the container changing when the cup is 1/3 of its total volume?

7. A triangular prismatic horse trough is leaking at a rate of 3 cubic feet per hour. The triangular sides have a base of 4 ft. The trough has a total depth of 3 feet and an overall length of 8 feet. At what time is the trough $^2/_3$ full?

What is the rate of change of the depth of water in the trough at this time?

8. For $4y^3 + 12x^2y - 24x^2 + 12y = -1$,

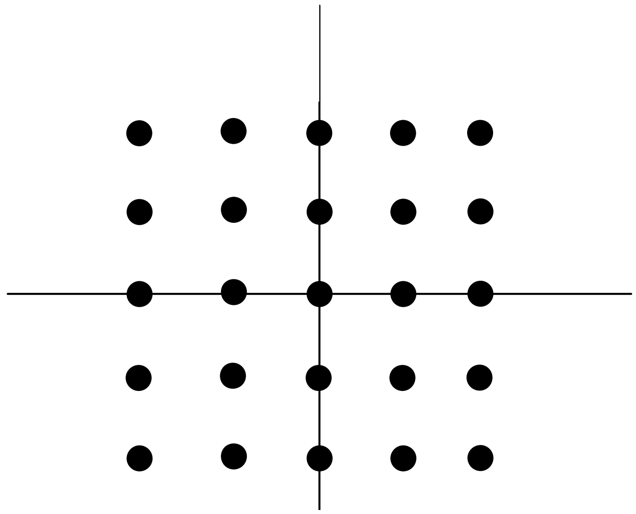
a) find $^{dy}/_{dx}$.

b) write the equation of each horizontal tangent.
(Show all computation and justify your answer.)

c) The line through the origin with slope of 1 is tangent to the curve at P. Find the x and y coordinates of P.
(Show all computation and justify your answer.)

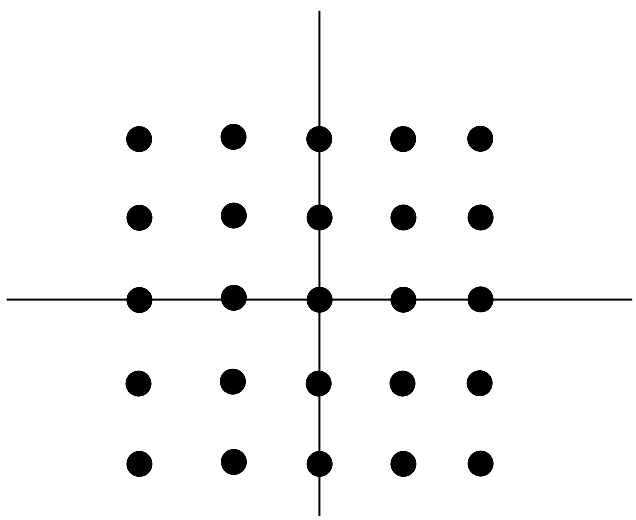
For problems 9 and 10, sketch a graph of the slope fields from the given information.

9. $\frac{dy}{dx} = -x$



Sketch a solution curve that passes through (0, -2).

10. $\frac{dy}{dx} = x - y$



Sketch a solution curve that passes through (-1, 1).

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AP Calculus ~ Semester I Review
Part 3

All problems are to be worked without a calculator unless otherwise noted.

Velocity and Acceleration

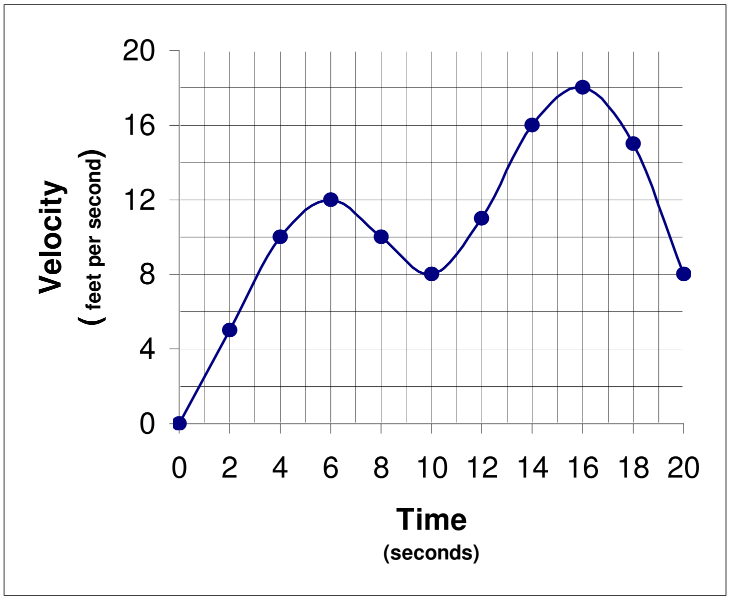
1. A particle moves along the x-axis so that its position at time t, is given by $x(t) = t^2 - 8t + 7$.
   For what values of t is the velocity zero?

2. Given $v(t) = -t^3 + 4t^2 + 3t - 5$.

a) Find the maximum velocity $0 \le t \le 4$

b) Find the maximum acceleration on the interval, $0 \le t \le 4$.

3. Use the graph and the table below to answer the following:
(Usage of a calculator is allowed on this problem.)



t (seconds)	**v(t) (ft per second)**
0	0
2	5
4	10
6	12
8	10
10	8
12	11
14	16
16	18
18	15
20	8

a) During what intervals is acceleration negative? *(Justify.)*

b) Find the average acceleration in ft/s^2 for $0 \le t \le 12$.
(Show your computation and justify your answer.)

c) Approximate the acceleration in ft/s^2 at t = 8.
(Show your computation and justify your answer.)

For problems 4-6, use calculus to answer each question. Show all work on separate sheet of paper. A calculator may be necessary.

4. A particle moves along the x-axis so that its position is given by $s(t) = 3 \cos t$ for $0 \leq t < 6.28$.

a) Find the position of the particle at 3 seconds.

b) List intervals on which the particle is to the left of the origin.

c) Find a function that describes the velocity of the particle.

d) Find the velocity of the particle at 3 seconds.

e) List intervals on which the function is moving to the right.

f) What is the average velocity of the particle between $t = 1$ and $t = 4$?

g) At what time is the instantaneous velocity of the particle equal to this average?

h) Find a function that describes the acceleration of the particle.

i) Find the acceleration of the particle at 3 seconds.

j) List intervals on which the particle is decelerating.

k) Summarize the particles position and movement at $t = 3$.

5. A particle moves along the x-axis so that its position is given by $s(t) = x^3 - x^2 - 6x$ in cm per second for $0 \leq t < 15$ seconds.

a) Find the position of the particle at 12 seconds.

b) List intervals on which the particle is to the right of the origin.

c) Find a function that describes the velocity of the particle.

d) Find the velocity of the particle at 12 seconds.

e) List intervals on which the function is moving to the left.

f) Find a function that describes the acceleration of the particle.

g) Find the acceleration of the particle at 12 seconds.

h) List intervals on which the particle is decelerating.

i) Summarize the particles position and movement at $t = 12$.

6. An object is thrown vertically upward at a rate of 5 ft per second, from the top of a 200 ft tower.
a) Write a function that describes its position.

b) What is the average velocity of the function on [1, 4]?

c) Find the maximum height and the time at which it occurs.

d) At what time will the object strike the ground?

e) Find a function that describes the velocity of the object.

f) At what time, on the closed interval of the problem situation will the object reach its maximum velocity? What is its maximum velocity?

g) What initial velocity would be necessary for the object to reach a maximum height of 500 feet?

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AP Calculus ~ Semester I Review
Part 4
Graphing Concepts

1. Given: f is continuous $a \le x \le b$ and differentiable $a < x < b$

a) f '(c)=

b) Under what circumstances must there be an extrema on $a < x < b$? *(Justify.)*

c) Must there be an absolute maximum or minimum on $a \le x \le b$? *(Justify.)*

d) Under what conditions must there be a c, where $a < c < b$, such that f '(c) = 0?

2. Find an equation in slope intercept form for the line tangent to the graph of **y = x – sin x** at the point $(-\pi, -\pi)$.

3. Find the x coordinate of the point of inflection for $y = \frac{1}{4}x^3 + 6x^2 + 32$.

4. If $f''(x) = x(x+1)^3(x-2)^2$ then the graph of f has inflection points at x = ?

5. On what intervals is the function $f(x) = x^3 + 2x^2 - 4x$ increasing?

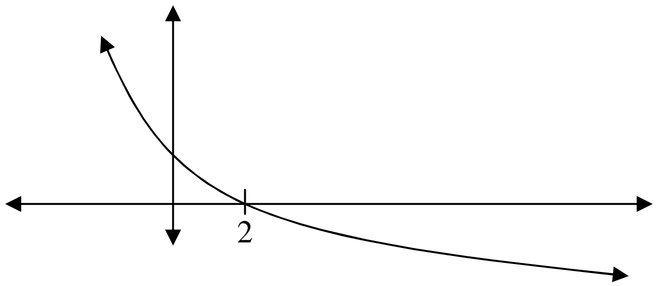
6. On what intervals is the function $f(x) = -x^4 - x^2 + 4$ concave down?

7. Find all local extrema by the First Derivative test for
$f(x) = x^3 - 27x$.

8. Find all local extrema by the Second Derivative test for
$f(x) = x^3 + 4x^2 + 4x - 16$.

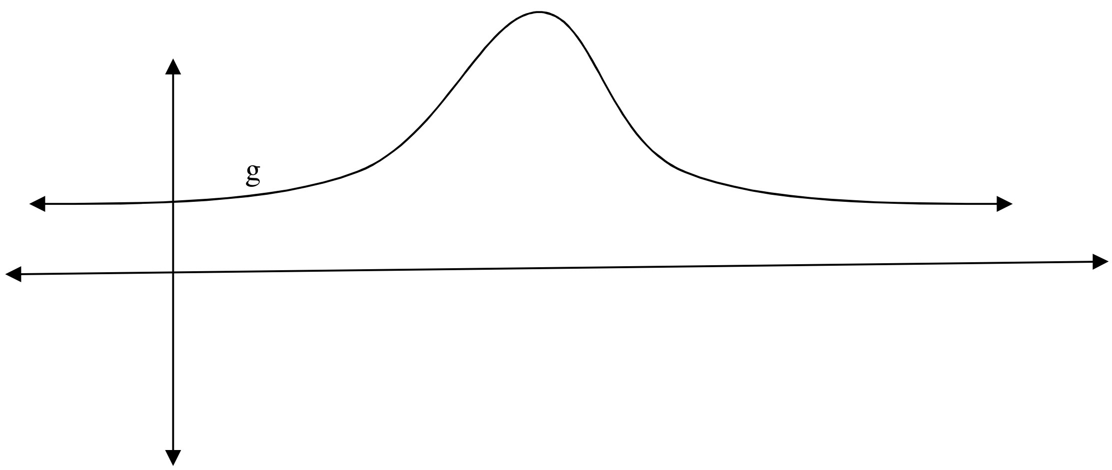
9. On what intervals is the function $f(x) = -x^4 - x^2 + 4$ increasing?

10.

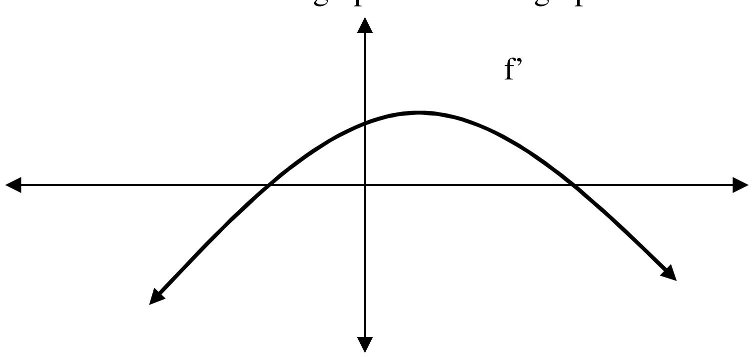


Arrange f(2), f '(2), and f "(2) in order from least to greatest. _______<_______<_______

11. Sketch and label an accurate graph of g 'on the graph of g.



12. Sketch and label an accurate graph of f on the graph of f'.



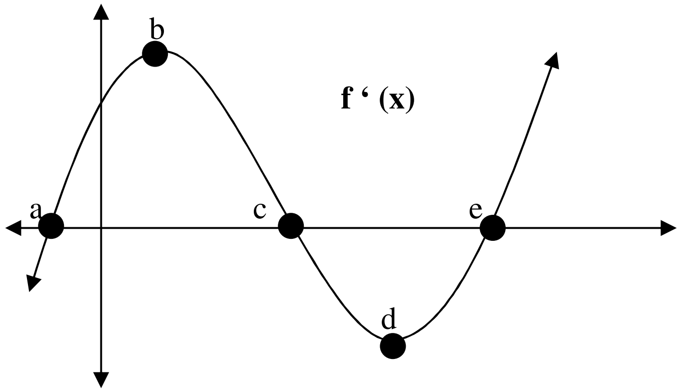
13. The function f, is continuous [-1, 3] and has values given in the table below.

x	-1	1	3
f(x)	-2	k	-4

What values of k make it possible that f(x) = 1 would have at least two solutions [-1, 3]?

(Usage of a calculator is allowed on the remainder of the problems.)

14.



a) Identify increasing and decreasing intervals for **f** based on the graph of **f '** on the interval **(a, e)**.

b) Identify x values for all relative extrema on **f** based on the graph of **f '** on the interval **(-∞, ∞)**.

c) Identify positive and negative intervals for **f ''** based on the graph of **f '** on the interval **(-∞, ∞)**.

15. True or false? Is the graph of $f(x) = |1 - x|$,

a) Continuous at x = 1?

b) Differentiable at x = 1?

c) Absolute min. at x = 1?

16. The first derivative of f is $f'(x) = \frac{1}{4} - \frac{\sin^2 x}{x}$. How many critical values does f have on (0, 5)?

17. Write the slope intercept form of the equation to the line tangent to the graph of $g(x) = x^4 - 2x^2$ at the point where g '(x) = 1 on the interval (0, 2).

18. If g is differentiable and $g(x) > 0$ for all x's and if $f'(x) = (9 - x^2)\, g(x)$, find the x coordinates for all extrema and identify each as a minimum or a maximum.

19. Let f be a function that is differentiable on the open interval (0,8). If f(1)= 4, f(3) = -5, and f(6) = 4:

a) How many zeros must f have? (*Justify.)*

b) Must the graph of f have any horizontal tangents? (*Justify.)*

c) For some c, $0< c <8$, must $f'(c)= -4$? (*Justify.)*

20. Let f be a function with f(2) = 6 such that for all points (x, y) on the graph of f the slope is given by $\dfrac{2x^2+1}{3y}$.

a) Find the slope of the graph of f at the point where x = 2.

b) Write an equation for the line tangent to the graph of f at x = 2.

c) Use the equation from b to approximate f (2.1).

21. Find the values of a and b if $f(x)= 3x^4 + ax^2 + b$ and f(x) has a relative maximum at (0, 2) and an inflection point where x = 1.