Limits-Trigonometric Functions

When calculating trig limits remember to consider the following:

1. Remember to still plug in the x- value first. If you get a numerical answer, that is your limit.
2. If you get $\frac{0}{0}$ after plugging in the x-value, that means there is a hole, and like the other problems with holes, there is a limit. Because we cannot factor, cancel and plug the x-value into what the function left after factoring, we will look at several of these situations to see if there is a pattern to these limits that we can use.
3. If you get $\frac{1}{0}$ or $\frac{\#}{0}$, then there is a vertical asymptote at that x-value and the limit DNE. All of the basic trig have asymptotes that go in opposite directions around the vertical asymptote. We will not pursue which direction the graph is heading with trig limits unless the function is squared or absolute valued.

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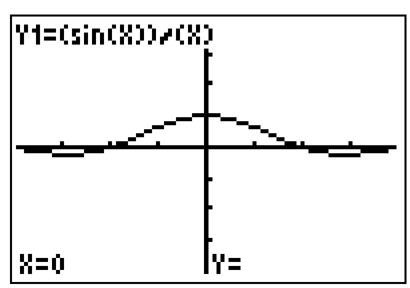
1. $\lim\limits_{x \to \frac{5\pi}{6}} \sin x =$ _____
2. $\lim\limits_{x \to -\frac{3\pi}{4}} \sin x =$ _____
3. $\lim\limits_{x \to \frac{2\pi}{3}} \cos x =$ _____
4. $\lim\limits_{x \to \frac{11\pi}{6}} \cos x =$ _____
5. $\lim\limits_{x \to \frac{2\pi}{3}} \sin 6x =$ _____
6. $\lim\limits_{x \to \frac{5\pi}{6}} (\sin 2x - \cos 6x) =$ _____
7. $\lim\limits_{x \to \frac{5\pi}{4}} 7\cos x =$ _____
8. $\lim\limits_{x \to -\frac{\pi}{4}} \csc x =$ _____
9. $\lim\limits_{x \to \frac{4\pi}{3}} \csc x =$ _____
10. $\lim\limits_{x \to \frac{\pi}{3}} \sec x =$ _____
11. $\lim\limits_{x \to \frac{7\pi}{6}} \sec x =$ _____
12. a. $\lim\limits_{x \to \frac{\pi}{3}} \tan x =$ _____
13. $\lim\limits_{x \to \frac{5\pi}{4}} \tan x =$ _____
14. a. $\lim\limits_{x \to \frac{5\pi}{6}} \cot x =$ _____
15. $\lim\limits_{x \to \frac{5\pi}{3}} \cot x =$ _____

Let's take a look at the following limit:



Example 1 $\lim\limits_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$

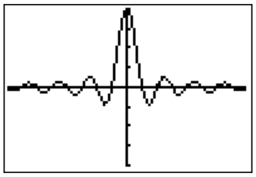
If 0 is plugged into the x of the function. We know that this means that there is a hole in the graph! Let's take a look at the picture and see if we can determine the y-value of the hole, which is the limit that we are looking for.

It appears that the y-values are approaching 1. $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

Now examine the graphs of the following functions and give their limits:

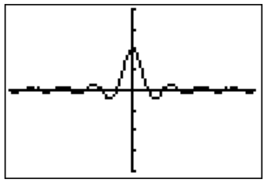
Example 2

$\lim\limits_{x\to 0}\dfrac{\sin 4x}{x} =$ ______



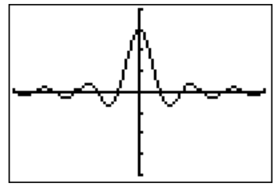
Example 3

$\lim\limits_{x\to 0}\dfrac{\sin 4x}{2x} =$ ____



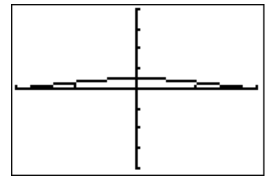
Example 4

$\lim\limits_{x\to 0}\dfrac{\sin 6x}{2x} =$ ____



Example 5

$\lim\limits_{x\to 0}\dfrac{\sin x}{2x} =$ ____

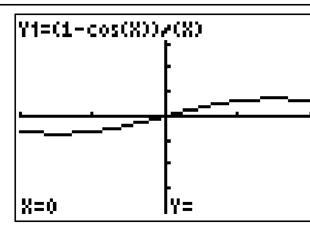


What do you notice about the coefficients of the x's and the answer to the limit? $\lim\limits_{x\to 0}\dfrac{\sin ax}{bx} = \dfrac{\quad}{\quad}$

Does this Pattern work for cos x in a similar pattern?

$\lim\limits_{x\to 0}\dfrac{\cos 3x}{6x} = \dfrac{1}{0}$ This means that there is a vertical asymptote on this graph at x=0. Always remember to plug in the x-value first!

Because $\tan x = \dfrac{\sin x}{\cos x}$ the pattern also works for tanx. $\lim\limits_{x\to 0}\dfrac{\tan 5x}{3x} = \dfrac{\quad}{\quad}$



There is one other famous trig function that does not show up a lot, but here it is: $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x} = \dfrac{0}{0}$, which means there is a hole and means that there is a limit. We could use something called the squeeze theorem, but we will just look at the graph and determine the answer.

After looking at the graph, $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x} = 0$ *or* $\lim\limits_{x\to 0}\dfrac{\cos x - 1}{x} = 0$.

Mixed Problems: Remember- The first thing you always do is……Plug in the number!

1. $\lim_{x\to 0}(2\sin x+3\cos x)$

2. $\lim_{x\to 0}\frac{x^2+1}{1-\sin x}$

3. $\lim_{x\to 0}\frac{x}{\tan x}$

4. $\lim_{x\to 0}\frac{x}{\sin 3x}$

5. $\lim_{x\to 0}\frac{\tan 2x}{x}$

Split this one up first and take the limit of each piece and add together.

6. $\lim_{x\to 0}\frac{x+\sin x}{x}$

Split this one up first and take the limit of each piece and multiply together.

7. $\lim_{x\to 0}\frac{\tan^2 x}{2x}=$

$\frac{\tan x\cdot\tan x}{2x}=\frac{\tan x}{2x}\cdot\frac{\tan x}{1}$

8. $\lim_{x\to 0}\frac{\tan 5x}{\tan 2x}$

9. $\lim_{x\to 0}3x\sec x$

10. $\lim_{x\to 0}\frac{\cos x-1}{x}$

11. $\lim_{x\to 0}\frac{\sin 3x+\tan 5x}{x}$

Be Careful- Remember the Golden Rule of Limits!

12. $\lim_{x\to 0}\frac{\tan 2x}{3}$