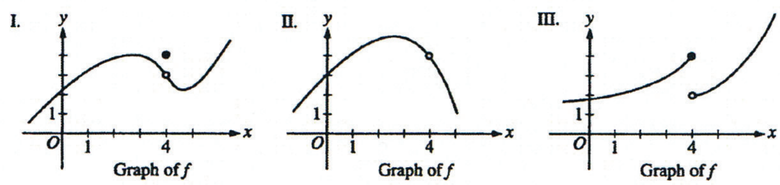
## Limits, Continuity and Derivatives For A+ College Ready Prep Sessions

### Multiple Choice

*Identify the choice that best completes the statement or answers the question.*

1. For which of the following does $\lim_{x \to 4} f(x)$ exist?




a. I only
b. II only
c. III only
d. I and II only
e. I and III only

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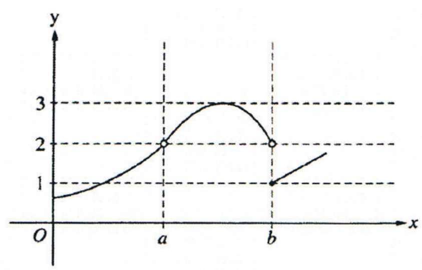
2.




The graph of the function $f$ is shown in the figure above. Which of the following statements about $f$ is true?

a. $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$
b. $\lim_{x \to a} f(x) = 2$
c. $\lim_{x \to b} f(x) = 2$
d. $\lim_{x \to b} f(x) = 1$
e. $\lim_{x \to a} f(x)$ does not exist

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3. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ the $\lim_{x \to 2} f(x)$ is

a. $\ln 2$
b. $\ln 8$
c. $\ln 16$
d. 4
e. nonexistent

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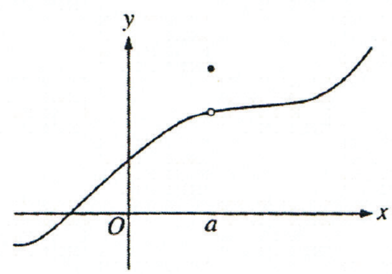


4. The graph of a function $f$ is shown above. Which of the following statements about $f$ is false?

a. $f$ is continuous at $x = a$.
b. $f$ has a relative maximum at $x = a$.
c. $x = a$ is in the domain of $f$.
d. $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$
e. $\lim_{x \to a} f(x)$ exists.

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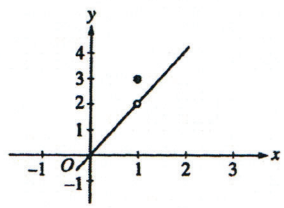
5. $\lim_{x \to 1} \dfrac{x}{\ln x}$ is

a. 0
b. $\dfrac{1}{e}$
c. 1
d. $e$
e. nonexistent

6.




Graph of $f$

The graph of the function $f$ is shown in the figure above. The value of $\lim_{x \to 1} \sin(f(x))$ is

a. 0.909
b. 0.841
c. 0.141
d. -0.416
e. nonexistent

7. If the function $f$ is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 4}{x + 2}$ when $x \neq 2$, then $f(-2)$=

a. -4
b. -2
c. -1
d. 0
e. 2

8. $\lim_{x \to \infty} \dfrac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

a. 4
b. 1
c. $\dfrac{1}{4}$
d. 0
e. -1

9. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function $f$ Which of the following statements must be true?

a. $f(0) = 2$
b. $f(x) \neq 2$ for all $x \geq 0$
c. $f(2)$ is undefined
d. $\lim_{x \to 2} f(x) = \infty$
e. $\lim_{x \to \infty} f(x) = 2$

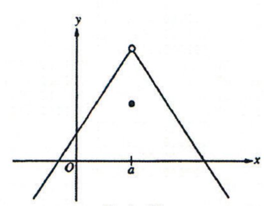
10. $\lim_{x \to 0} (x \csc x)$ is

a. $-\infty$
b. -1
c. 0
d. 1
e. $\infty$

11. If $\lim_{x \to a} f(x) = L$, where $L$ is a real number, which of the following must be true?

a. $f'(a)$ exists.
b. $f(x)$ is continuous at $x = a$.
c. $f(x)$ is defined at $x = a$.
d. $f(a) = L$
e. None of the above

12.




Graph of $f$

The graph of the function $f$ is shown above. Which of the following statements must be false?

a. $f(a)$ exists.
b. $f(x)$ is defined for $0 < x < a$.
c. $f$ is not continuous at $x = a$.
d. $\lim_{x \to a} f(x)$ exists.
e. $\lim_{x \to a} f'(x)$ exists

13. $\lim_{\theta \to 0} \frac{1 - \cos\theta}{2\sin^2\theta}$ is

a. $0$
b. $\frac{1}{8}$
c. $\frac{1}{4}$
d. $1$
e. nonexistent

14. Let $f$ be a function such that $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 5$.
Which of the following must be true?
I. $f$ is continuous at x = 2.
II. $f$ is differentiable at x = 2.
III. The derivative of $f$ is continuous at x = 2.

a. I only
b. II only
c. I and II only
d. I and III only
e. II and III only

15.

$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

Let $f$ be the function given above. Which of the following statements are true about $f$?

I. $\lim_{x \to 3} f(x)$ exists
II. $f$ is continuous at $x = 3$.
III. $f$ is differentiable at $x = 3$.

a. None
b. I only
c. II only
d. I and II only
e. I, II, and III

16. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

a. $\frac{3x-3}{\sqrt{2x-3}}$
b. $\frac{x}{\sqrt{2x-3}}$
c. $\frac{1}{\sqrt{2x-3}}$
d. $\frac{-x+3}{\sqrt{2x-3}}$
e. $\frac{5x-6}{2\sqrt{2x-3}}$

17. If $f(x) = \frac{e^{2x}}{2x}$, then $f'(x) =$

a. $1$
b. $\frac{e^{2x}(1-2x)}{2x^2}$
c. $e^{2x}$
d. $\frac{e^{2x}(2x+1)}{x^2}$
e. $\frac{e^{2x}(2x-1)}{2x^2}$

18. If $y = \left(x^3+1\right)^2$, then $\frac{dy}{dx} =$

a. $\left(3x^2\right)^2$
b. $2\left(x^3+1\right)$
c. $2\left(3x^2+1\right)$
d. $3x^2\left(x^3+1\right)$
e. $6x^2\left(x^3+1\right)$

19. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

a. $-\cos\left(e^{-x}\right)$

b. $\cos\left(e^{-x}\right) + e^{-x}$

c. $\cos\left(e^{-x}\right) - e^{-x}$

d. $e^{-x}\cos\left(e^{-x}\right)$

e. $-e^{-x}\cos\left(e^{-x}\right)$

20. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

a. $-\frac{2}{5}$

b. $\frac{1}{5}$

c. $\frac{1}{4}$

d. $\frac{2}{5}$

e. nonexistent

21. $\frac{d}{dx}\cos^2\left(x^3\right) =$

a. $6x^2\sin\left(x^3\right)\cos\left(x^3\right)$

b. $6x^2\cos\left(x^3\right)$

c. $\sin^2\left(x^3\right)$

d. $-6x^2\sin\left(x^3\right)\cos\left(x^3\right)$

e. $-2\sin\left(x^3\right)\cos\left(x^3\right)$

22. If $f(x) = \cos(3x)$, then $f'(\frac{\pi}{9}) =$

a. $\frac{3\sqrt{3}}{2}$

b. $\frac{\sqrt{3}}{2}$

c. $-\frac{\sqrt{3}}{2}$

d. $-\frac{3}{2}$

e. $-\frac{3\sqrt{3}}{2}$

23.

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

Let $f$ be the function defined above, where $c$ and $d$ are constants. If f is differentiable at $x = 2$, what is the value of $c + d$ ?

a. $-4$
b. $-2$
c. 0
d. 2
e. 4

24. Let $f$ and $g$ be differentiable functions with the following properties:

(i) $g(x) > 0$ for all $x$
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

a. $f'(x)$
b. $g(x)$
c. $e^x$
d. 0
e. 1

25.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| –1 | 6 | 5 | 3 | –2 |
| 1 | 3 | –3 | –1 | 2 |
| 3 | 1 | –2 | 2 | 3 |

The table above gives values of $f$, $f'$, $g$, and $g'$ at selected values of $x$. If $h(x) = f(g(x))$, then $h'(1) =$

a. 5
b. 6
c. 9
d. 10
e. 12

26. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

a. $\lim\limits_{h \to 0} \dfrac{e^{x+h}}{h}$

b. $\lim\limits_{h \to 0} \dfrac{e^{x+h} - e^e}{h}$

c. $\lim\limits_{h \to 0} \dfrac{e^{e+h} - e}{h}$

d. $\lim\limits_{h \to 0} \dfrac{e^{x+h} - 1}{h}$

e. $\lim\limits_{h \to 0} \dfrac{e^{e+h} - e^e}{h}$

27. If $y = 2\cos\left(\dfrac{x}{2}\right)$, then $\dfrac{d^2y}{dx^2} =$

a. $-8\cos\left(\dfrac{x}{2}\right)$

b. $-2\cos\left(\dfrac{x}{2}\right)$

c. $-\sin\left(\dfrac{x}{2}\right)$

d. $-\cos\left(\dfrac{x}{2}\right)$

e. $-\dfrac{1}{2}\cos\left(\dfrac{x}{2}\right)$

# GRAPHIC ORGANIZER FOR BIG THEOREMS

Hypotheses

INTERMEDIATE VALUE THEOREM

Conculusions

Picture

Hypotheses

**Mean Value Theorem**

Conculusions

Picture

Hypotheses

EXTREME VALUE THEOREM

Conculusions

Picture

CANDIDATES' TEST