2010 AB 2 d only

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

2011 AB B1 Part D

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

What does f'(x) and f''(x) tell us about the original function f(x)?

Fill out the charts for a continuous f(x)

THE FIRST DERIVATIVE

The derivative of f(x)	The original function f(x)	What could the graph of f(x) looks like
f'(x) is positive f'(x)>0	f(x)	
f'(x) is negative f'(x)<0	f(x)	
f'(x) is zero f'(x)=0	f(x)	
	f(x)	
	f(x)	
f'(x) is undefined	f(x)	
	f(x)	
	f(x)	
Critical Point		

THE SECOND DERIVATIVE

The second derivative of f(x)	The original function f(x)	What the graph of f(x) looks like
f''(x) is positive f''(x)>0	f(x)	
f''(x) is negative f''(x)<0	f(x)	
f''(x) is zero f''(x)=0	f(x)	

The First Derivative graph is given, what does that tell me about the original?

The derivative graph of f(x)	f'(x) is	The original function f(x)
The derivative graph is above the x-axis	f'(x)	f(x)
The derivative graph is below the x-axis	f'(x)	f(x)
The derivative graph crosses the x-axis and goes from above the axis to below the axis	f'(x)	f(x)
The derivative graph crosses the x-axis and goes from below the axis to above the axis	f'(x)	f(x)
The derivative graph touches but does not cross through the x-axis	f'(x)	f(x)

THE FIRST DERIVATIVE IS … MEANS THE SECOND DERIVATIVE IS…WHICH MEANS THE ORIGINAL IS …

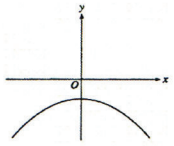
The derivative of f(x)	The second derivative of f(x)	The original function f(x)
f'(x) increasing	f''(x)	f(x)
f'(x) is decreasing	f''(x)	f(x)
f'(x) has a max	f''(x)	f(x)
f'(x) has a min	f''(x)	f(x)
f'(x) has a horizontal layout	f''(x)	f(x)

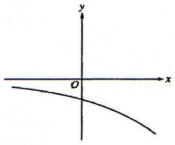
f, f',f'' Multiple Choice

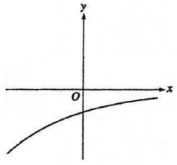
Multiple Choice

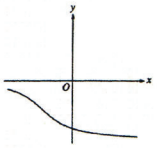
Identify the choice that best completes the statement or answers the question.

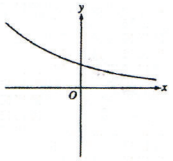
1. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f?

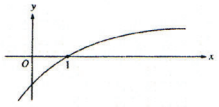
a. 

b. 

c. 

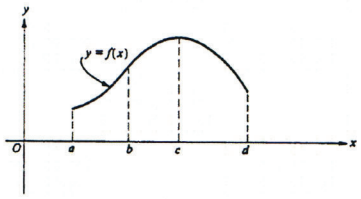
d. 

e. 

2. 

The graph of a twice-differentiable function f is shown In the figure above. Which of the following is true?

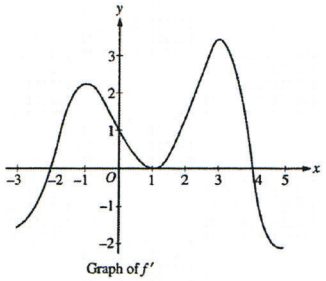
a. $f(1) < f'(1) < f''(1)$
b. $f(1) < f''(1) < f'(1)$
c. $f'(1) < f(1) < f''(1)$
d. $f''(1) < f(1) < f'(1)$
e. $f''(1) < f'(1) < f(1)$

3. 

The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

a. I only
b. II only
c. III only
d. I and II
e. II and III

4. 

Graph of f'

The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

a. -2 only
b. 1 only
c. 4 only
d. -1 and 3 only
e. -2, 1, and 4

5. **GC** The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \le x \le 2$. On what intervals is f increasing?

   a. $1 \le x \le 1.445$ only
   b. $1 \le x \le 1.691$
   c. $1.445 \le x \le 1.875$
   d. $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$
   e. $0 \le x \le 1$ and $1.691 \le x \le 2$

6. **GC** The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval (0, 10)?

   a. One
   b. Three
   c. Four
   d. Five
   e. Seven

7. **GC** If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

   a. -0.46
   b. 0.20
   c. 0.91
   d. 0.95
   e. 3.73

8. **GC** Aparticle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time t =4?

   a. -2.016
   b. -0.677
   c. 1.633
   d. 1.814
   e. 2.978

9. **GC** The function f has fIrst derivative given by $f'(x) = \frac{\sqrt{x}}{1 + x + x^3}$ What is the x-coordinate ofthe inflection point of the graph of f?

   a. 1.008
   b. 0.473
   c. 0
   d. -0.278
   e. The graph of f has no inflection point.

10. **GC** The derivative of the function f is given by $f'(x) = x^2\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

    a. One
    b. Two
    c. Three
    d. Four
    e. Five

11. What are all values of x for which the function f defined by $f(x) = \left(x^2 - 3\right)e^{-x}$ is increasing?

    a. There are no such values of x.
    b. $x < -1$ and $x > 3$
    c. $-3 < x < 1$
    d. $-1 < x < 3$
    e. All values of x

12. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

    a. $(-\infty, -1]$ only
    b. $(-\infty, 0)$
    c. $[-1, 0)$ only
    d. $(0, \sqrt[3]{2}\,]$
    e. $[\sqrt[3]{2}, \infty)$

13. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

a. f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
b. f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
c. f has relative minima at $x = -2$ and at $x = 2$.
d. f has relative maxima at $x = -2$ and at $x = 2$.
e. It cannot be determined if f has any relative extrema.

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14. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

a. $x < 0$
b. $x > 0$
c. $x < -2$ or $x > -\frac{2}{3}$
d. $x < \frac{2}{3}$ or $x > 2$
e. $\frac{2}{3} < x < 2$

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15. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

a. $x < -2$
b. $x > -2$
c. $x < -1$
d. $x > -2$
e. $x < 0$

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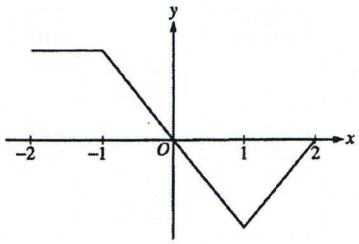
16. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

a. -1 only
b. 2 only
c. -1 and 0 only
d. -1 and 2 only
e. -1,0, and 2 only

17.



Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

a. f is decreasing for $-1 \le x \le 1$.
b. f is increasing for $-2 \le x \le 0$.
c. f is increasing for $1 \le x \le 2$.
d. f has a local minimum at $x = 0$.
e. f is not differentiable at $x = -1$ and $x = 1$.

18.

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

a. $-2 \le x \le 2$ only
b. $-1 \le x \le 1$ only
c. $x \ge -2$
d. $x \ge 2$ only
e. $x \le -2$ or $x \ge 2$

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19. A particle moves along the *x-axis* so that at time $t \ge 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

a. $t = 1$ only
b. $t = 3$ only
c. $t = \frac{7}{2}$ only
d. $t = 3$ and $t = \frac{7}{2}$
e. $t = 3$ and $t = 4$