

Calculus 1 Worksheet #4

Limits involving trigonometric functions: $\lim_{x \rightarrow 0} \frac{\sin(\square)}{\square}$

KNOW THE FOLLOWING THREE THEOREMS:

A. $\lim_{x \rightarrow 0} \frac{\sin \square}{\square} = 1$	B. $\lim_{x \rightarrow 0} \frac{\square}{\sin \square} = 1$	C. $\lim_{x \rightarrow 0} \frac{1 - \cos \square}{\square} = 0$
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Examples:

1. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \left[\frac{3}{3} \right] \Rightarrow \lim_{x \rightarrow 0} 3 \left[\frac{\sin 3x}{3x} \right] = \boxed{3}$
2. $\lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x} \cdot \left[\frac{7}{7} \right] \Rightarrow \lim_{x \rightarrow 0} 7 \left[\frac{1 - \cos 7x}{7x} \right] = \boxed{0}$
3. $\lim_{x \rightarrow 0} \frac{\tan 2x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{\cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{x \cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{x \cos 2x} \cdot \left[\frac{2}{2} \right] \Rightarrow$ $\lim_{x \rightarrow 0} \frac{2}{\cos 2x} \left[\frac{\sin 2x}{2x} \right] \Rightarrow \lim_{x \rightarrow 0} \frac{2}{\cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{2}{\cos 2(0)} = \boxed{2}$

Problems:

1. $\lim_{x \rightarrow 0} \frac{\sin \frac{1}{2}x}{x}$	2. $\lim_{x \rightarrow 0} x \csc x$	3. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin x}$	4. $\lim_{x \rightarrow 0} \frac{\sin ax}{x}, a \neq 0$
5. $\lim_{x \rightarrow 0} \frac{\tan x}{x}$	6. $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x}$	7. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$	8. $\lim_{x \rightarrow 0} \frac{\sin x}{2x}$
9. $\lim_{x \rightarrow 0} \frac{3 \sin x}{x}$	10. $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$	11. $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x}$	12. $\lim_{x \rightarrow 0} \frac{3x}{\sin x}$
13. $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$	14. $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$	15. $\lim_{x \rightarrow 0} \frac{\sin^4 2x}{4x^4}$	16. $\lim_{x \rightarrow 0} \frac{\sin 5x}{5x}$
17. $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{2x}$	18. $\lim_{x \rightarrow 0} \frac{x+2}{\cos x}$	19. $\lim_{x \rightarrow \frac{\pi}{4}} (\tan x)$	20. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$

Answers:

1) $\frac{1}{2}$	2) 1	3) 2	4) a	5) 1	6) $\frac{3}{2}$	7) 3	8) $\frac{1}{2}$
9) 3	10) $\frac{3}{5}$	11) 2	12) 3	13) 0	14) $\frac{a}{b}$	15) 4	16) 1
17) 0	18) 2	19) 1	20) $\frac{1}{2}$				