Exam 2 - Part I – 28 questions – No Calculator Allowed

1. If $f(x)=\ln\left(\frac{e}{x^n}\right)$, and n is a constant, then $f'(x)=$

A. $\frac{-n}{x}$ B. $\frac{x^n}{e}$ C. $\frac{-1}{x^n}$ D. $\frac{e}{x^n}$ E. 0

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2. Let f be the function defined below. Which of the following statements about f are **NOT** true?

$$f(x)=\begin{cases}\frac{x^3-1}{x-1}, x\neq 1\\ 3x, x=1\end{cases}$$

I. f has a limit at $x=1$.
II. f is continuous at $x=1$.
III. f is differentiable at $x=1$.
IV. The derivative of f' is continuous at $x=1$.

A. IV only
B. III and IV only
C. II, III, and IV only
D. I, II, III, and IV
E. All statements are true

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3. The absolute maximum value of $y=\left|x^2-x-12\right|$ on $-4\leq x\leq 4$ is

A. 0.5 B. 4 C. 8 D. 11.75 E. 12.25

4. Find the equation of the tangent line to $x^3 + y^3 = 3xy + 5x - 3y$ at $(2,2)$.

A. $x + 9y = 20$
B. $13x - 3y = 20$
C. $9y - x = 16$
D. $y = \frac{-1}{9}$
E. $y = 2$

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5. Find $\lim_{x \to -\infty} \frac{(3x-1)(x^2-4)}{(2x+1)^2(x-1)}$

A. $-\frac{3}{2}$
B. $\frac{3}{2}$
C. $\frac{3}{4}$
D. 1
E. ∞

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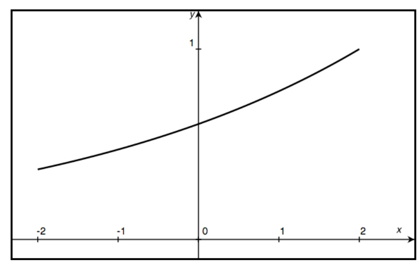
6. The graph of the function f is shown to the right for $-2 \le x \le 2$. Four calculations are made:



LS – Left Riemann sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

RS – Right Riemann sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

TS – Trapezoidal sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

DI – $\int_{-2}^{2} f(x)\,dx$

Arrange the results of the calculations from highest to lowest.

A. DI – RS – TS - LS
B. RS – TS – DI - LS
C. TS – DI – LS - RS
D. LS – DI – TS - RS
E. RS – DI – TS - LS

7. What is the equation of the line normal to the graph of $y = 2\sin x\cos x - \sin x$ at $x = \frac{3\pi}{2}$?

A. $y = 2x - 3\pi + 1$ B. $y = -2x + 3\pi + 1$ C. $y = \frac{x}{2} - \frac{3\pi}{4} + 1$

D. $y = -\frac{x}{2} + \frac{3\pi}{4} + 1$ E. $y = -\frac{x}{2} + \frac{3\pi}{4} - 1$

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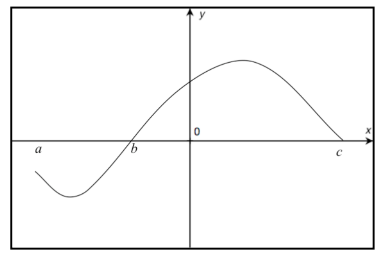
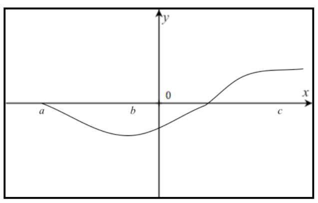
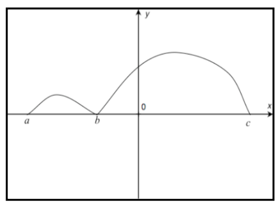
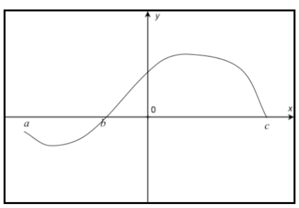
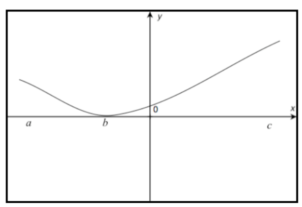
8. Let $f(x) = \int_a^x g(t)\,dt$ where g has the graph shown to the right. Which of the following could be the graph of f?

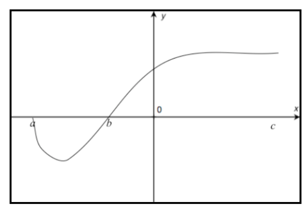


A. 

B. 

C. 

D. 

E. 

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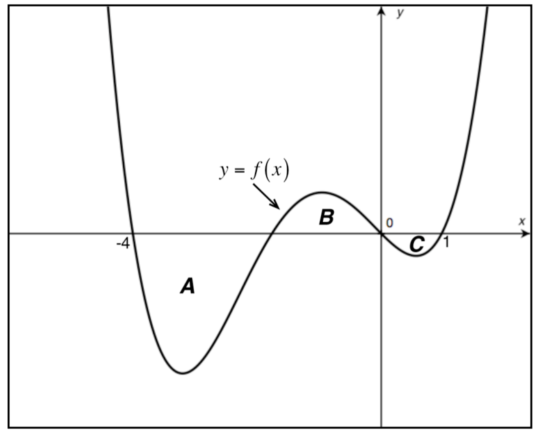
9. The rate of change of the volume V of the oil in a tank with respect to time t is inversely proportional to the square root of V. If the tank is empty at time $t = 0$ and $V = 9$ when $t = 2$, find the equation which describes the relationship between t and V.

A. $V = \frac{9}{4}t^2$ B. $V = \frac{t^2}{36}$ C. $V^{3/2} = \frac{4}{9}t$ D. $V^{3/2} = \frac{27}{2}t$ E. $V = 3t$

10. The three regions A, B, and C in the figure to the right are bounded by the graph of the function f and the x-axis. If the areas of A, B, and C are 5, 2, and 1 respectively, what is the value of $\int_{-4}^{1} [f(x)+2]\, dx$?



A. 28
B. 10
C. 6
D. -4
E. -12

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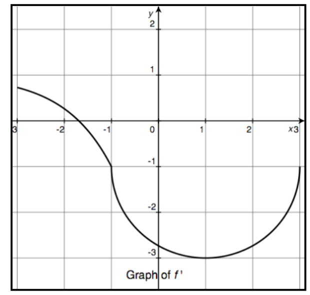
11. The table below gives value of the differentiable function f and g at $x=-1$. If $h(x)=\dfrac{f(x)-g(x)}{2f(x)}$, then $h'(-1)=$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-2	4	e	-3

A. $\dfrac{-e-3}{4}$ B. $\dfrac{e+3}{2e}$ C. $\dfrac{e-6}{8}$ D. $\dfrac{2e-3}{4}$ E. $\dfrac{-4e-3}{4}$

12. The derivative of a function f is defined by $f'(x)=\begin{cases} g(x), & -1\le x\le 1 \\ 1-2e^{x+1}, & -3\le x<-1 \end{cases}$.

The graph of the continuous function f' is shown to the right with the graph of g being a semicircle. Which of the following statements are true?



I. f has a relative maximum at $x=-1-\ln 2$
II. f has an inflection point at $x = 1$
III. f is increasing on (1,3)

A. I only B. II only C. I and II only D. II and III only E. I, II and III

The number of vehicles waiting to get past an accident scene is modeled by a twice-differentiable N of t, where t is measured in minutes. For $0 < t < 8$, $N''(t) > 0$. The table below gives selected values of the rate of change $N'(t)$ over the time interval $0 \le t \le 8$. The number of vehicles in line at $t = 4$ is 45. By using the tangent line approximation at $t = 4$, which of the following is a possible value for the number of vehicles in line at $t = 4.5$ minutes?

t (minutes)	0	1	4	6	8
$N'(t)$ (vehicles per minutes)	7	10	24	32	42

I. 57 II. 59 III. 63

A. I only B. II only C. III only D. II and III only E. I, II and III

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14. If $f(x)=\tan^{-1}(2x)$, find $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$.

A. $\frac{2}{17}$ B. $\frac{1}{17}$ C. $\frac{-2}{15}$ D. $\frac{-1}{15}$ E. nonexistent

15. The table below gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval [-4, 4], what is the value of $\int_{-2}^{2} f'(2x)\,dx$?

x	-4	-2	-1	1	2	4
$f(x)$	8	-6	-4	-2	-4	4
$f'(x)$	-6	-4	-2	0	2	4

A. -2 B. -8 C. 5 D. 20 E. 1

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16. If $f(x) = k\sin x + \ln x$ where k is a constant has a critical point at $x = \pi$, determine which statement below is true.

A. f has a relative minimum at $x = \pi$
B. f has a relative maximum at $x = \pi$
C. f has a point of inflection at $x = \pi$
D. f has no concavity at $x = \pi$
E. None of the above

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17. How many horizontal (H) and vertical (V) tangent lines does the graph of $(x^2 + 4)y = 8$ have?

A. 2 H, 1V B. 2 H, 0V C. 1 H, 0V D. 0 H, 2V E. 0 H, 0V

18. A particle moving along the x-axis such that at any time $t \geq 0$ its position is given by $x(t) = 2 - 30t + 81t^2 - 20t^3$. For what values of t is the particle moving to the left?

A. $0 \leq t < \frac{27}{20}$
B. $t > \frac{27}{20}$
C. $0 \leq t < \frac{1}{5}, t > \frac{5}{2}$
D. $\frac{1}{5} < t < \frac{5}{2}$
E. $t > \frac{1}{5}$

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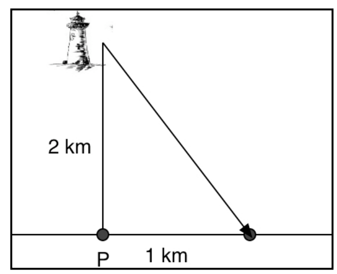
19. A lighthouse is 2 km from a point P along a straight shoreline and its light makes 1 revolution per minute. How fast, in km/min, is the beam of light moving along the shoreline when it is 1 km from point P?



A. 2.5
B. 10π
C. 20π
D. 5π
E. 10

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20. Let f be the function given by $f(x) = \int_x^{\pi/2} t \sin 2t \, dt$ for $-\pi \leq x \leq \pi$. In what interval(s) is $f(x)$ decreasing?

A. $(0, \pi)$
B. $\left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)$
C. $\left(-\pi, \frac{-\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$
D. $(-\pi, 0)$
E. $(-\pi, \pi)$

21. If the region bounded by the y-axis, the line $y = 4$, and the curve $y = \sqrt{x}$ is revolved about the x-axis, the volume of the solid generated would be

A. $\frac{88\pi}{3}$ B. 56π C. 128π D. $\frac{128\pi}{3}$ E. $\frac{640\pi}{3}$

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22. The function f is twice differentiable with $f(-3) = -2$ and $f'(-3) = -4$. For what value of c is the approximation of $f(c)$ using the tangent line of f at $x = c$ equal to c?

A. –4.667 B. –0.4 C. –2.8 D. –2.5 E. –3.333

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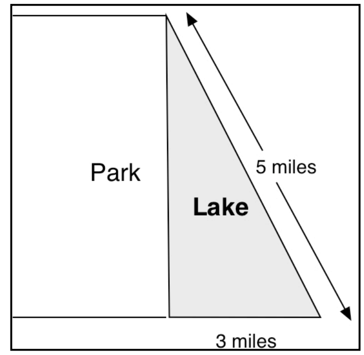
23. Let f be a differentiable function such that $f(-4) = 12$, $f(9) = -4$, $f'(4) = -6$, $f'(9) = 3$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(-4)$?

A. $\frac{-1}{6}$ B. $\frac{-1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{9}$ E. Insufficient data

24. A lake shaped in a right triangle is located next to a park as shown in the figure to the right. The depth of the lake at any point along a strip x miles from the park's edge is $f(x)$ feet. Which of the following expressions gives the total volume of the lake?



A. $\int_0^3 \left(4-\frac{4}{3}x\right) f(x)\,dx$ B. $\int_0^4 \left(3-\frac{3}{4}x\right) f(x)\,dx$ C. $2\int_0^3 f(x)\,dx$

D. $\int_0^4 f(x)\,dx$ E. $\frac{3}{2}\int_0^4 f(x)\,dx$

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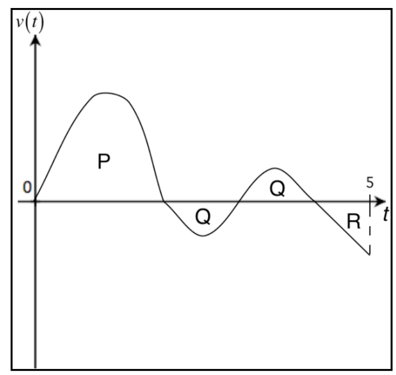
25. A particle moves along the y-axis with velocity $v(t)$ for $0 \le t \le 5$. The graph of $v(t)$ is shown to the right with the positive areas indicated by P, Q, and R. What is the difference between the distance that the particle travels and its displacement for $0 \le t \le 5$?



A. P - R B. 2(P + Q + R) C. 2Q

D. 2R E. 2(Q + R)

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26. $\int_1^x \frac{\ln t}{t}\,dt =$

A. $\frac{x^2-1}{2}$ B. $\frac{x^2}{2}$ C. $\frac{(\ln x)^2}{2}$ D. $\frac{(\ln x)^2-1}{2}$ E. $\ln x$

27. $\int \frac{e^{\sqrt{x+2}}}{\sqrt{8x+16}}\,dx =$

A. $2e^{\sqrt{x+2}}+C$

B. $\frac{e^{\sqrt{x+2}}}{2}+C$

C. $\frac{e^{\sqrt{x+2}}}{8}+C$

D. $\sqrt{2}e^{\sqrt{x+2}}+C$

E. $\frac{\sqrt{2}e^{\sqrt{x+2}}}{2}+C$

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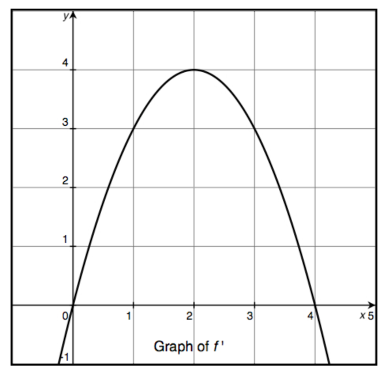
28. The function g is defined by $g(x)=x^2-f(x)$ where the graph of $f'(x)$ is pictured to the right. Find and classify all points of relative extrema of $g(x)$.



A. Relative maximum at $x = 2$ only
B. Relative minimum at $x = 2$ only
C. Relative maximum at $x = 2$ and relative minimum at $x = 0$
D. Relative maximum at $x = 0$ and relative minimum at $x = 2$
E. Relative minimum at $x = 0$ and relative maximum at $x = 4$

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29. Let $f'(x)=(x-1)^2\cos(x^2+1)$. If c represents the largest value on the x-axis on [0, 2] where there is a point of relative extrema of $f(x)$, and let d represents the smallest value on the x-axis on [0, 2] where there is a point of relative extrema of $f(x)$, find the value of $c-d$.

A. 0.244 B. 0.927 C. 1.171
D. -0.689 E. there are no relative extrema

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30. A particle moving along the x-axis such that at any time $t \geq 0$ its velocity is given by $v(t)=\dfrac{t^3}{\ln\sqrt{t}}$. What is the acceleration of the particle at $t = e$?

A. $6e^2-2e$ B. $6e^3$ C. $2e^3$ D. $4e^2$ E. $6e^2-4e^{5/2}$

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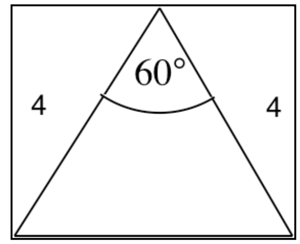
31. If $\sin(\pi x)\cos(\pi y)=y$, find $\dfrac{dy}{dx}$ at $(1,0)$

A. $1+\pi$ B. $1-\pi$ C. -1 D. $-\pi$ E. π

32. An isosceles triangle with its two equal sides of 4 inches has the angle between them changing at the rate of $10°/\text{min}$. How fast is the 3rd side of the triangle changing, in inches/min, when the angle between the two given sides is $60°$?



A. 0.605 B. 17.321 C. 34.641
D. 1.209 E. 8.660

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33. If $a \neq 0$ and n is a positive integer, then $\lim_{x \to a} \frac{x^n - a^n}{x^{2n} - a^{2n}}$ is

A. $\frac{1}{a^n}$ B. $\frac{1}{2a^n}$ C. $\frac{1}{a^{2n}}$ D. 0 E. nonexistent

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34. A cup of coffee is heated to boiling $(212°\text{F})$ and taken out of a microwave and placed in a $72°\text{F}$ room at time $t = 0$ minutes. The coffee cools at the rate of $16e^{-0.112t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the coffee at time $t = 5$ minutes?

A. 105°F B. 133°F C. 166°F D. 151°F E. 203°F

35. What is the average value of $y=\dfrac{\sin(2x+3)}{x^2+2x+3}$ on the interval [-2, 3] ?

A. -0.057 B. 0.051 C. 0.061 D. 0.256 E. 0.303

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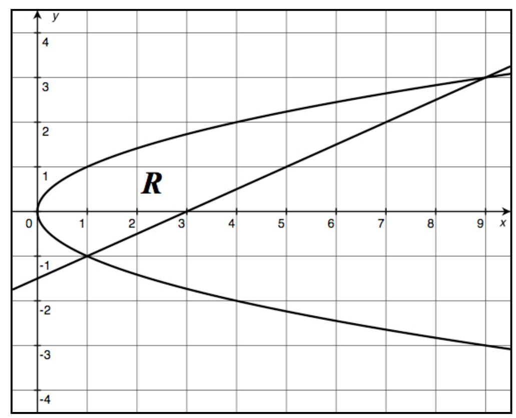
36. Region R, enclosed by the graphs of $y=\pm\sqrt{x}$ and $y=\dfrac{x-3}{2}$ is shown in the figure to the right. Which of the following calculations would accurately compute the area of region R?



I. $\displaystyle\int_0^9\left(\sqrt{x}-\frac{x-3}{2}\right)dx$

II. $\displaystyle\int_0^1 2\sqrt{x}\,dx+\int_1^9\left(\sqrt{x}-\frac{x-3}{2}\right)dx$

III. $\displaystyle\int_{-1}^3\left(2x+3-x^2\right)dx$

A. I only B. II only C. III only D. II and III E. I, II, and III

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37. A particle moves along the x-axis so that at any time $t\geq 0$, its velocity is given by $v(t)=5-2t-3t^2$. If the particle is at position $x=-3$ at time $t=2$, where was the particle at $t=1$?

A. -5 B. 2 C. 3 D. 4 E. 11

38. The function f is defined for $x > 0$ with $f(\pi) = 1$, and $f'(x) = \sin\left(\frac{1}{x}\right)$. Use the line tangent to f at $x = \pi$ to approximate the value of $f(4)$ and whether it over or under-approximates $f(4)$.

A. 1.269 – under-estimates
B. 1.269 – over-estimates
C. 1.815 – under-estimates
D. 1.815 – over-estimates
E. 0.731 – over-estimates

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39. Find the derivative of $f^{-1}(x)$ for $f(x) = x^3 - 3x^2 - 2x + 1$ at $x = 2$.

A. 0.064
B. –0.500
C. 3.627
D. 0.005
E. -0.045

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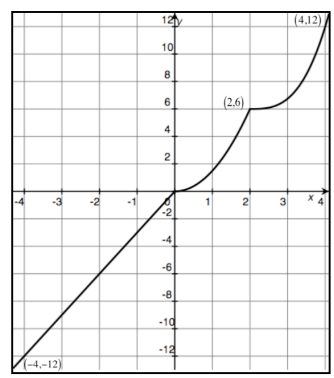
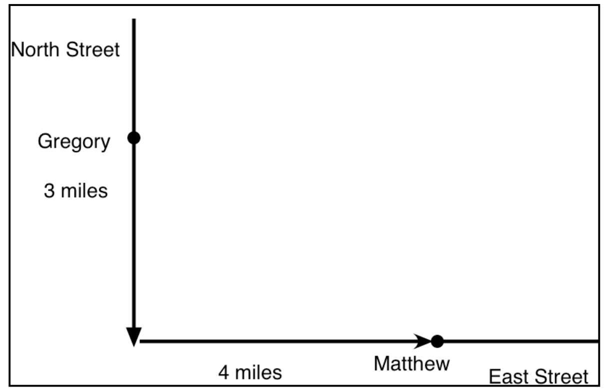
40. A continuous function f is defined on the closed interval $-4 \leq x \leq 4$. The graph of the function, shown in the figure to the right consists of a line and two curves. There is a value a, $-4 \leq a < 4$, for which the Mean Value Theorem, applied to the interval $[a, 4]$ guarantees a value c, $a \leq c < 4$ at which $f'(c) = 3$. What are possible values of a?



I. -4 II. 0 III. 2

A I only
B. II only
C. III only
D. II and III only
E. I, II, and III

41. Matthew is visiting Gregory at his home on North Street. Shortly after Matthew leaves, Gregory realizes that Matthew left his wallet and begins to chase him. When Gregory is 3 miles from the 90° intersection along North Street traveling at 40 mph towards the intersection, Matthew is 4 miles along East street traveling away from the intersection at 25 mph. At that time, how fast is the distance between the two men changing?



A. getting closer at 4 mph
B. getting further away at 44 mph
C. getting closer at 44 mph
D. Getting closer at 5 mph
E. getting closer at 7 mph

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42. Let f be the function with the first derivative defined as $f'(x) = \sin(x^3 - 4x + 2)$ for $0 \le x \le 2$. Let c be the value of x where f attains its minimum value in the interval $0 \le x \le 2$ and let d be the value of x where f attains its maximum value in the interval $0 \le x \le 2$. Find $|c - d|$.

A. 0.539
B. 1.136
C. 1.461
D. 1.675
E. 2

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43. If f is a continuous function and $F'(x) = f(x)$ for all real numbers x, then $\int_{-2}^{2} f(1 - 3x)\, dx =$

A. $3F(-2) - 3F(2)$
B. $\frac{1}{3}F(-2) - \frac{1}{3}3F(2)$
C. $\frac{1}{3}F(2) - \frac{1}{3}3F(-2)$
D. $3F(-5) - 3F(7)$
E. $\frac{1}{3}F(7) - \frac{1}{3}F(-5)$

44. Let $f(x) = 3x^2 - 6x$. For how many positive values of b does the average value of $f(x)$ on the interval [0, b] equal the average rate of change of $f(x)$ on the interval [0, b] ?

A. 4 B. 3 C. 2 D. 1 E. none

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45. If $\frac{dy}{dx} = 3x^2y^2$ and if $y = 1$ when $x = 2$, then when $x = -1$, $y =$

A. $\frac{1}{10}$ B. $\frac{1}{8}$ C. $\frac{1}{19}$ D. $\frac{-1}{8}$ E. -1