Part I – 28 questions – No Calculator Allowed

1. If $f(x)=\sqrt[3]{\cos^2(3x)}$, then $f'(x)=$

A. $\dfrac{-2}{3\sqrt[3]{\sin 3x}}$ B. $\dfrac{-2}{\sqrt[3]{\sin 3x}}$ C. $\dfrac{-2\sin 3x}{\sqrt[3]{\cos 3x}}$ D. $\dfrac{2}{\sqrt[3]{\cos 3x}}$ E. $\dfrac{-2\sin 3x}{3\sqrt[3]{\cos 3x}}$

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2. $\int_1^3 \sqrt{20-4x}\,dx$ is equivalent to

A. $\dfrac{-1}{4}\int_{-1/5}^{1/5}\sqrt{u}\,du$ B. $\dfrac{-1}{4}\int_1^3\sqrt{u}\,du$ C. $\dfrac{1}{4}\int_8^{16}\sqrt{u}\,du$ D. $4\int_8^{16}\sqrt{u}\,du$ E. $\int_8^{16}\sqrt{u}\,du$

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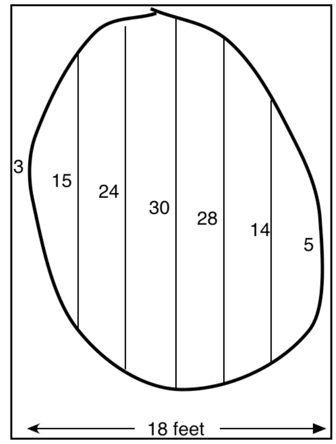
3. Let *f* be the function defined below, where *c* and *d* are constants. If *f* is differentiable at $x=-1$, what is the value of $c-d$?

$$f(x)=\begin{cases} x^2+(2c+1)x-d, x\ge -1 \\ e^{2x+2}+cx+3d, x<-1 \end{cases}$$

A. -2 B. 0 C. 2 D. 3 E. 4

4. An oil slick forms in a lake. The oil is 4 feet deep. The slick is 18 feet wide and is divided into 6 sections of equal width having measurements as shown in the figure to the right. Tim uses a trapezoidal sum with six trapezoids to approximate the volume while Doug uses a midpoint sum using three equally spaced rectangles. What is the difference between their approximations of the volume of the slick in cubic feet?



A. 9 B. 36
C. 12 D. 3
E. 0

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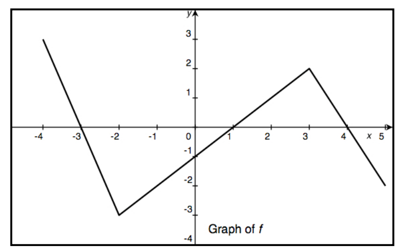
5. The graph of the piecewise linear function f is shown in the figure to right. If $g(x)=\int_{1}^{x} f(t)\,dt$, which of the following is the greatest?



A. $g(-4)$ B. $g(-3)$ C. $g(1)$
D. $g(4)$ E. $g(5)$

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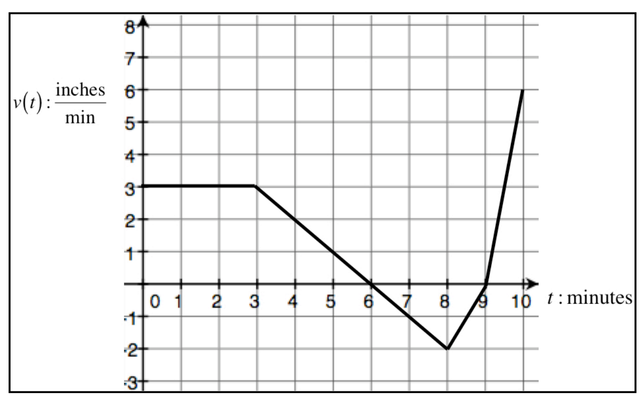
6. A spider is walking up a vertical wall. At $t = 0$, the spider is 18 inches off the floor. The velocity v of the spider, measured in inches per minute at time t minutes, $0 \le t \le 10$ is given by the function whose graph is shown to the right. How many inches off the floor is the spider at $t = 10$?



A. 13.5 B. 19.5 C. 24
D. 31.5 E. 37.5

7. The velocity of a particle moving along the x-axis is given by the function $v(t) = te^{t^2}$. What is the average velocity of the particle from time $t = 1$ to $t = 3$?

A. $\dfrac{19e^9 - 3e}{2}$ B. $4e^9$ C. $\dfrac{3e^9 - e}{2}$ D. $\dfrac{e^9 - e}{4}$ E. $e^9 - e$

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8. $\lim\limits_{h \to 0} \dfrac{\sin(\pi + h) - \sin \pi}{h} =$

A. 0 B. $\cos x$ C. -1 D. π E. 1

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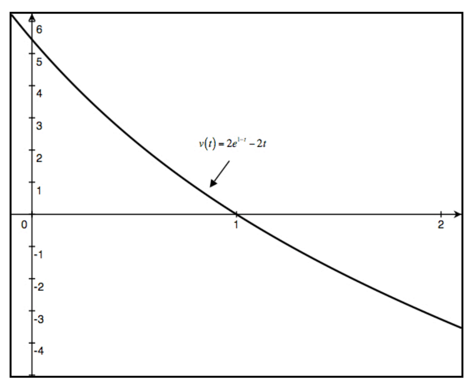
9. A particle moves along the x-axis so that its velocity $v(t) = 2e^{1-t} - 2t$, as shown in the figure to the right. Find the total distance that the particle traveled from $t = 0$ to $t = 2$.



A. $2e + \dfrac{2}{e} - 2$ B. $2e - \dfrac{2}{e} - 4$

C. $2e - \dfrac{2}{e} + 4$ D. $2e - 2$

E. $2e + \dfrac{2}{e} + 2$

10. A new robotic dog called the IPup went on sale at (9 AM) and sold out within 8 hours. The number of customers in line to purchase the IPup at time t *is* modeled by a differentiable function A where $0 \le t \le 8$. Values of $A(t)$ are shown in the table below. For $0 \le t \le 8$, what is the fewest number of times at which $A'(t) = 0$?

t (hours)	0	1	2	3	4	5	6	7	8
$A(t)$ people	150	185	135	120	75	75	100	120	60

A. 0 B. 2 C. 3 D. 4 E. 5

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11. A particle moving along a straight line that at any time $t \ge 0$ its velocity is given by $v(t) = \dfrac{\sin t}{\cos 2t}$. For which values of t is the particle speeding up?

I. $t = \dfrac{\pi}{6}$ II. $t = \dfrac{\pi}{3}$ III. $t = \dfrac{2\pi}{3}$

A. I only B. II only C. III only D. I and II only E. I and III only

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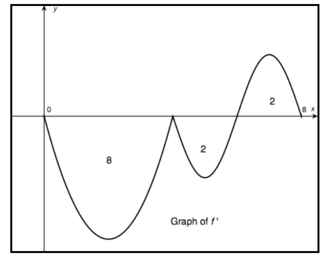
12. If $f'(x) = \dfrac{x^2 - 5x - 5}{e^{-x}}$, for what values of x is $f(x)$ concave down?

A. All values of x B. No values of x C. $-2 < x < 5$

D. $x > 5$ or $x < -2$ E. $\dfrac{1-\sqrt{53}}{2} < x < \dfrac{1+\sqrt{53}}{2}$

13. The graph of f', the derivative of f, is shown to the right for $0 \le x \le 8$. The areas of the regions between the graph of f' and the axis are 8, 2 and 2 respectively. If $f(0) = 10$, what is the minimum value of f on the interval $0 \le x \le 8$?



A. -12 B. -10 C. -8
D. 0 E. 10

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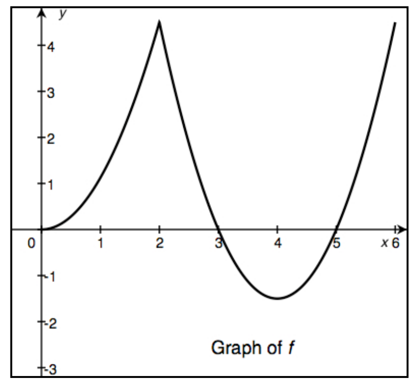
14. The graph of the function f shown to the right has a horizontal tangent at $x = 4$. Let g be the continuous function defined by $g(x) = \int_0^x f(t)\,dt$.
For what value(s) of x does the graph of g have a point of inflection?



A. 3 and 5 B. 0, 3, and 5 C. 2 only

D. 2 and 4 E. 4 only

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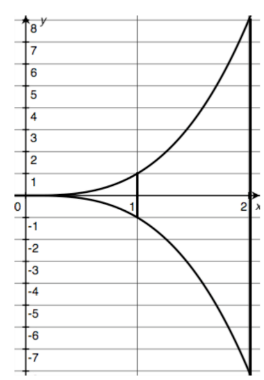
15. A piece of candy is determined by the curves $y = \pm x^3$, for $1 \le x \le 2$ as shown in the figure to the right. For this piece of candy, the cross sections perpendicular to the x-axis are equilateral triangles. What is the volume of the piece of candy in cubic units?



A. $\dfrac{127}{7}$ B. $\dfrac{15\sqrt{3}}{2}$ C. $\dfrac{254\sqrt{3}}{7}$
D. $\dfrac{127\sqrt{3}}{21}$ E. $\dfrac{127\sqrt{3}}{7}$

16. Let g be a strictly decreasing function such that $g(x)<0$ for all real numbers x. If $f(x)=(x-1)^2 g(x)$, which of the following is true?

A. f has a relative minimum at $x = 1$
B. f has a relative maximum at $x = 1$
C. f will be a strictly decreasing function
D. f will be a strictly increasing function
E. It cannot be determined if f has a relative extrema

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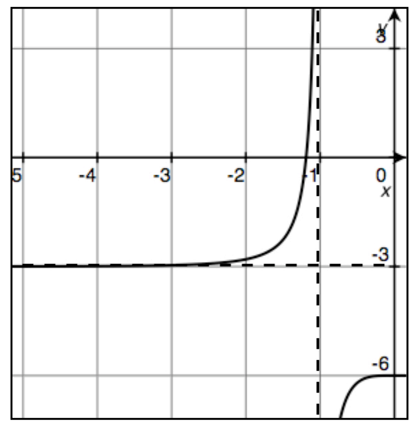
17. The function f is given by $f(x)=\dfrac{ax^4+6}{x^4+b}$. The figure to the right shows a portion of the graph of f. Which of the following could be the values of the constants a and b?



A. a = -3, b = -1
B. a = 3, b = 1
C. a = 3, b = -1
D. a = -3, b = 1
E. a = 6, b = -1

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18. $f(x)=e^{2\ln\tan(x^2)}$, then $f'(x)=$

A. $e^{2\ln\tan(x^2)}$ B. $\dfrac{e^{2\ln\tan(x^2)}}{x^2}$ C. $\dfrac{e^{2\ln\tan(x^2)}}{2\tan x^2}$ D. $\dfrac{2\sin(x^2)}{\cos^3(x^2)}$ E. $\dfrac{4x\sin(x^2)}{\cos^3(x^2)}$

19. If $f(x)=2^{x^2-x-1}$, find the equation of the tangent line to f at $x=-1$.

A. $y=2x+4$ B. $y=-6x-4$ C. $y=2-2\ln 2(x+1)$
D. $y=2-6\ln 2(x+1)$ E. $y=2-3\ln 2(x+1)$

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20. The function g is differentiable for all real numbers. The table below gives values of the function and its first derivatives at selected values of x. If g^{-1} is the inverse function of g, what is the equation for the line tangent to the graph of $y=g^{-1}(x)$ at $x=4$?

x	$g(x)$	$g'(x)$
-2	4	2
4	-3	5

A. $y+3=\frac{1}{5}(x-4)$ B. $y+2=\frac{1}{5}(x-4)$ C. $y+2=2(x-4)$
D. $y+3=\frac{1}{2}(x-4)$ E. $y+2=\frac{1}{2}(x-4)$

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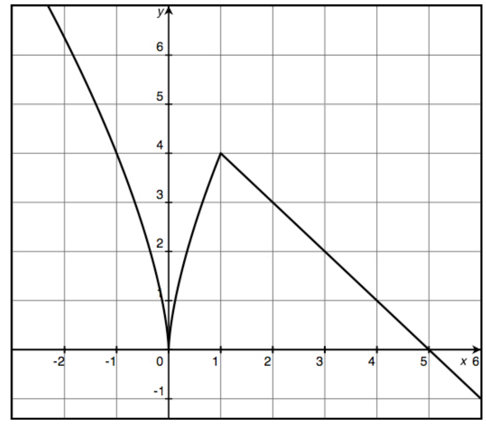
21. If $f'(x)=(2x-1)^2(3x+1)^3(x-1)$, then f has which of the following relative extrema?

I. A relative maximum at $x=\frac{-1}{3}$
II. A relative minimum at $x=\frac{1}{2}$
III. A relative minimum at $x=1$

A. I only B. III only C. I and II only D. I and III only E. I, II and III

22. The graph of $f(x)$ consists of a curve that is symmetric to the y-axis on [-1, 1] and a line segment as shown to the right. Which of the following statements about f is false?



A. $\lim_{x\to 0}[f(x)-f(0)]=0$

B. $\lim_{x\to 0}\frac{f(x)-f(0)}{x}=0$

C. $\lim_{h\to 0}\frac{f(0+h)-f(0-h)}{2h}=0$

D. $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=-1$

E. $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$ does not exist

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23. If $f(x)=\sin^3(4x)$, find $f'\left(\frac{\pi}{3}\right)$.

A. $-\frac{3}{2}$ B. $-\frac{9}{2}$ C. $-3\sqrt{3}$ D. $3\sqrt{3}$ E. $-\frac{9}{8}$

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24. Let x and y be functions of time t that are related by the equation $y^2=xy+e$. At time $t=1$, the value of y is e and $\frac{dy}{dt}=2$. Find $\frac{dx}{dt}$ when $t=1$.

A. $\frac{2e-2}{e}$ B. $\frac{2e+2}{e}$ C. $\frac{2}{e}$ D. $\frac{e+2}{e}$ E. $\frac{e-2}{e}$

25. Let $f(x)$ be given by the function below. What values of a, b, and c do **NOT** make $f(x)$ differentiable?

$$f(x)=\begin{cases} a\cos x, x\leq 0 \\ b\sin(x+c\pi), x>0 \end{cases}$$

A. $a=0,\ b=0,\ c=0$

B. $a=0,\ b=0,\ c=100$

C. $a=5,\ b=5,\ c=0.5$

D. $a=-1,\ b=1,\ c=1$

E. $a=-8,\ b=8,\ c=-2.5$

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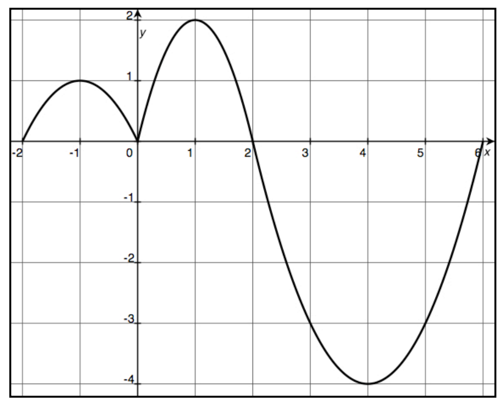
26. The graph of f', the derivative of f is shown to the right for $-2 \leq x < 6$. At what values of x does f have a horizontal tangent line?



A. $x = 0$ only
B. $x = -1, x = 1, x = 4$
C. $x = 2$ only
D. $x = -2, x = 2, x = 6$
E. $x = -2, x = 0, x = 2, x = 6$

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27. A sphere with radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$. At a certain instant the rate of decrease of its volume is twice the rate of decrease of its surface area. What is the radius of the sphere at that point in time?

A. $\frac{1}{4}$

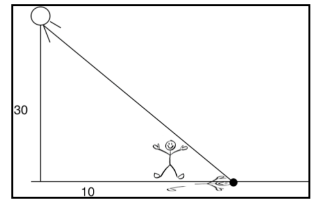
B. $\frac{1}{2}$

C. 1

D. 2

E. 4

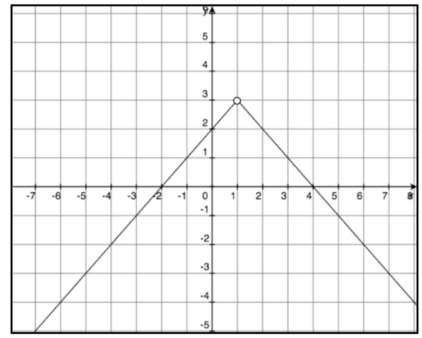
28. Frank, who is 6 feet tall, is performing along a bare stage as shown in the figure to the right. There is a single spotlight 30 feet above the left wing of the stage and Frank is walking across the stage from left to right at 3 feet/sec. When Frank is 10 feet from the left edge of the stage, how fast is the tip of his shadow moving across the stage?



A. 15 ft/sec B. $\frac{3}{4} \text{ ft/sec}$ C. $\frac{15}{4} \text{ ft/sec}$ D. $\frac{3}{5} \text{ ft/sec}$ E. $\frac{18}{5} \text{ ft/sec}$

Part II – 17 questions – Calculators Allowed

29. The figure to the right shows the graph of $f(x)$.
Which of the following statements are true?



I. $\lim_{x \to 1^-} f(x)$ exists

II. $\lim_{x \to 1^+} f(x)$ exists

III. $\lim_{x \to 1} f(x)$ exists

A. I only B. II only C. I and II only D. I, II and III E. none are true

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30. There are value(s) of c that satisfy the Mean-Value Theorem for $f(x) = 2\cos x - 4\cos 2x$ on $[0, \pi]$. Find the sum of these values.

A. 1.455 B. 4.493 C. 4.596 D. 1.687 E. -1.273

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31. A tube is in the shape of a right circular cylinder. The height is increasing at the rate of 1.5 inches/sec while the radius is decreasing that the rate of 0.5 inches/sec. At the time that the radius is 4 inches and the length is 14 inches, how is the lateral surface area of the tube changing? (The lateral surface area of a right circular cylinder with radius r and height h is $2\pi rh$).

A. Decreasing at $1\ \text{in}^2/\text{sec}$ B. Increasing at $1\ \text{in}^2/\text{sec}$ C. Decreasing at $2\pi\ \text{in}^2/\text{sec}$
D. Increasing at $26\pi\ \text{in}^2/\text{sec}$ E. Increasing at $38\pi\ \text{in}^2/\text{sec}$

32. Cellophane-wrapped Tastycakes come off the assembly line at the rate of $C(t)=5+4\sin\left(\frac{\pi t}{6}\right)$. They are boxed at the rate of $B(t)=\frac{20t}{1+t^2}$. $C(t)$ and $B(t)$ are measured in hundreds of cakes per hour, and t is measured in hours for $0 \le t \le 5$. There are 500 cakes waiting to be boxed at the start of the 5-hour shift. How long (in hours) does it take the number of cakes waiting to be boxed to go from a minimum to a maximum during the 5-hour shift?

A. 1.560 B. 1.869 C. 3.131 D. 4.691 E. 5

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33. Consider the differential equation $\frac{dy}{dx}=x^2-2y+3$. If $y=f(x)$ is the solution to the differential equation, at what point does f have a relative minimum?

A. (0, 0) B. (-1, 2) C. (4, 9) D. $\left(0,\frac{3}{2}\right)$ E. (5, 14)

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34. If $x^2+y^2=a^2$, where a is a constant, find $\frac{d^2y}{dx^2}$.

A. $\frac{-y^2-x^2}{y^2}$ B. $\frac{x^2-y^2}{y^3}$ C. $\frac{-1}{y^2}$ D. $\frac{-a^2}{y^3}$ E. $\frac{1}{a^2}$

35. Let f be the function given by $f(x)=\dfrac{4x^2}{e^x - e^{-x}}$. For what positive value(s) of c is $f'(c)=1$?

A. 1.122
B. 0.824 and 4.306
C. 0.258 and 3.260
D. 3.264
E. 0.523 and 4.307

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36. If $y=\csc^{-1}\left(x^2\right)$, which of the following represents $\dfrac{dy}{dx}$?

A. $2x\sin x^2$
B. $-2x\sin y\tan y$
C. $\dfrac{2x}{\cos y}$
D. $\dfrac{-2x}{\sin^2 y}$
E. $\dfrac{-2x}{\sin^2\left(x^2\right)}$

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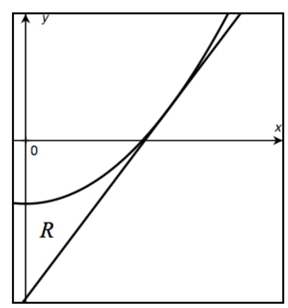
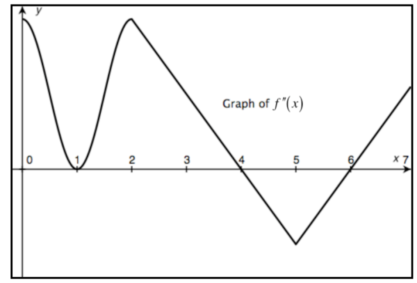
37. On the graph to the right, the line $y=kx-10$, k a constant, is tangent to the graph of $y=x^2-4$. Region R is enclosed by the graphs of $y=kx-10$, $y=x^2-4$, and the y-axis. Find the area of region R.



A. 4.287
B. 4.899
C. 6.124
D. 9.798
E. 12.247

38. The graph of f'', the second derivative of f, is shown to the right. On which of the following intervals is $f'(x)$ decreasing?



A. [0, 1]
B. [0, 1] and [2, 4]
C. [0, 1] and [2, 5]
D. [4, 5]
E. [4, 6]

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39. The Cheesesteak Factory at a Philadelphia stadium has 25 steak sandwiches ready to sell when it opens for business. It cooks cheesesteaks at the rate of 4 steaks per minute and sells cheesesteaks at the rate of $1+6\sin\left(\frac{2\pi t}{41}\right)$ steaks per minute. For $0 \le t \le 20$, at what time is the number of steaks ready to sell at a minimum (nearest tenth of a minute)?

A. 0
B. 3.4
C. 10.3
D. 17.1
E. 20

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40. Barstow, California has the daily summer temperature modeled by $T(t) = 85 + 27\sin\left[\frac{\pi(t-9)}{12}\right]$ where t is measured in hours and t = 0 corresponds to 12:00 midnight. What is the average temperature in Barstow during the period from 1:30 PM to 6 PM?

A. 60°
B. 89°
C. 104°
D. 110°
E. 115°

41. $\int_0^{\pi} e^{\cos x} \sin x \, dx =$

A. $\frac{1}{e}$ B. $e-\frac{1}{e}$ C. $e-1$ D. $1-\frac{1}{e}$ E. e

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42. $\int \frac{x+2}{x^2+4} \, dx =$

A. $\frac{\ln\left(x^2+4\right)+\tan^{-1}(x)}{2}+C$ B. $\tan^{-1}\left(\frac{x}{2}\right)+C$ C. $\ln(x+2)+C$

D. $\frac{1}{2}\ln\left(x^2+4\right)+\tan^{-1}\left(\frac{x}{2}\right)+C$ E. $\frac{1}{2}\left[\ln\left(x^2+4\right)+\tan^{-1}\left(\frac{x}{2}\right)\right]+C$

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43. If the region bounded by the x-axis, the line $x=\frac{\pi}{2}$, and the curve $y=\sin x$ is revolved about the line $x=\frac{\pi}{2}$, the volume of the solid generated would be

A. 1.142 B. 1.468 C. 1.793 D. 3.142 E. 3.586

44. Let $F(x)$ be an antiderivative of $\frac{(\ln x - 3)^2}{x}$. If $F(1) = 2$, then $F(2) =$

A. -11.092 B. -8.769 C. 2.092 D. 6.908 E. 16.724

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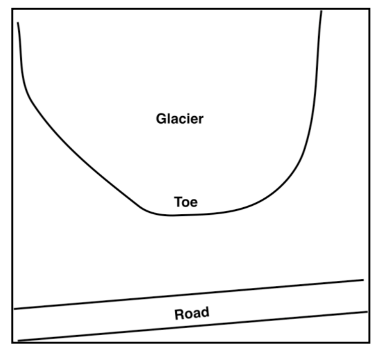
45. The Athabasca Glacier in Canada is one of the few glaciers to which you can drive. Like many glaciers in the world, it is receding (in this case its toe is moving away from the road). In the year 2000, its toe was 500 feet from the road and in the year 2010, its toe was 600 feet from the road. The rate the distance that the toe increases from the road is proportional to that distance. Assuming the glacier recedes at the same rate, what will be the average distance in feet per year (nearest integer) the glacier recedes from 2000 to the year 2050?



A. 10 B. 13 C. 15 D. 16 E. 18