Name______________________________ Class__________ Date______________

AP Calculus ~ Semester I Review

Part 1

All problems are to be worked without a calculator unless otherwise noted.

Limits

Determine the following limits by direct evaluation, factoring and reducing, L'Hoptital's Rule or the definition of a derivative.

1. $\lim_{x\to 1}\frac{x^3-1}{x-1} = \frac{0}{0}$ $\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}$

$1+1+1$

L'Hopitals or $\lim_{x\to 1}\frac{3x^2}{1}$

Answer: 3

2. $\lim_{x\to -3}\frac{x^2+x-6}{x+3} = \frac{9-3-6}{0} = \frac{0}{0}$

$\frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}}$

$-3-2$

Answer: -5

3. $\lim_{x\to\infty}\frac{\sqrt{4x^2+1}}{3x+2} = \frac{+2x}{+3x} = \frac{2}{3}$

Answer: $\frac{2}{3}$

4. $\lim_{x\to -2}\frac{x^3-2}{x-1}$ $\frac{-8-2}{-3} =$

Answer: $\frac{-10}{-3}$ or $\frac{10}{3}$

5. $\lim_{x\to 0}\frac{\sin \cancel{x}}{\cancel{x}} = \frac{1}{1}$

$\lim_{x\to 0}\frac{\sin ax}{bx} = \frac{a}{b}$

Answer: 1

6. $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$

Special trig limit Know this

Answer: 0

7. $\lim_{\Delta x\to 0}\frac{\left[(x+\Delta x)^3+2(x+\Delta x)\right]-\left[(x)^3+2(x)\right]}{\Delta x}$

Def of der $f(x) = x^3+2x$

Answer is $f'(x)$

Answer: $3x^2+2$

8. $\lim_{x\to 0}\frac{\sin 4x}{x} =$

Answer: 4

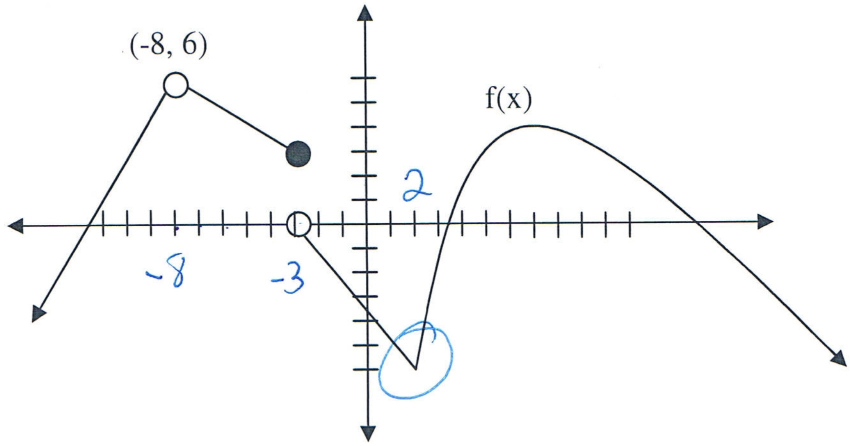
9. $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$

Def of Der $f(x) = \sin x$ at $x = \frac{\pi}{2}$

$f'(\frac{\pi}{2})$ is answer $f'(x) = \cos x$

$f'(\frac{\pi}{2}) = \cos\frac{\pi}{2}$

Answer: 0



Answer questions 10-14 from the above graph of f(x).

10. $\lim_{x\to -8} f(x)$ Hole 6

11. $\lim_{x\to -3} f(x)$ DNE

12. $\lim_{x\to -3^-} f(x)$ = 3

13. Name the continuous intervals for f(x).

$(-\infty,-8)\cup(-8,-3)\cup(-3,+\infty)$

14. Name the differentiable intervals for f(x).

$(-\infty,-8)\cup(-8,-3)\cup(-3,2)(2,+\infty)$

Derivatives

Product

15. Find f'(2) for $f(x)=(x^2-2x+1)(x^3-1)$.

$f'(x)=(x^2-2x+1)(3x^2)+(x^3-1)(2x-2)$

$f'(2)=(\cancel{4}-\cancel{4}+1)(12)+(7)(2)$

12 + 14

26

Quotient

16. Find g'(2) for $g(x)=\frac{3-2x-x^2}{x^2-1}$.

$g'(x)=\frac{(x^2-1)(-2-2x)-(3-2x-x^2)(2x)}{(x^2-1)^2}$

$g'(2)=\frac{(3)(-6)-(-5)(4)}{9}=\frac{2}{9}$

$\frac{2}{9}$

Chain

17. Find h'(3) for $h(x)=(3x-2x^2)^3$.

$h'(x)=3(3x-2x^2)^2(3-4x)$

$h'(3)=3(9-18)^2(3-12)$

$=3(81)(-9)$

$=(243)(-9)$

-2187

-2187

18. Find q'(x) for $q(x)=x^2\sqrt{1-x^2}$. Product / Chain

$q'(x)=x^2\frac{1}{2\sqrt{1-x^2}}\cdot -2x+\sqrt{1-x^2}(2x)$

Complete the following identities.

19. $d/_{dx} \tan x =$ $\sec^2 x$

20. $d/_{dx} \sec x =$ $\sec x \tan x$

21. $d/_{dx} \cot x =$ $-\csc^2 x$

22. $d/_{dx} \csc x =$ $-\csc x \cot x$

23. $d/_{dx} e^u =$ $e^u \frac{du}{dx}$

24. $d/_{dx} \cos^2(x^3) =$ $-6x^2 \cos(x^3) \sin(x^3)$

25. If $g(x) = \tan e^{-x}$, $g'(x) =$ $-e^{-x} \sec^2(e^{-x})$

$y = (\cos(x^3))^2$

$y' = 2\cos(x^3) \cdot -\sin(x^3)\ 3x^2$

26. If $f(x) = \tan(3x)$, find $f'(\frac{\pi}{9}) =$ 12

$f'(x) = \sec^2(3x)\ (3)$

$f'(\frac{\pi}{9}) = (\sec 3 \cdot \frac{\pi}{9})^2 (3) = \frac{1}{(\cos\frac{\pi}{3})^2} \cdot 3 = 4 \cdot 3$

27. If $f(x) = \cos e^{2x}$, find $f'(x) =$ $-2e^{2x} \sin e^{2x}$

$f'(x) = -\sin e^{2x} \cdot e^{2x} \cdot 2$

28. State continuous and differentiable intervals for $f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2 - 3x, & x \geq 1 \end{cases}$.

Cont $(-\infty, +\infty)$
Diff $(-\infty, 1) \cup (1, +\infty)$

① $3(1) - 4 = -1$
$2 - 3(1) = -1$ Cont ✓

② $6x$ at 1 6
-3 -3

29. If $f(x) = \begin{cases} x + k, & x \leq 2 \\ x^2 + 1, & x > 2 \end{cases}$, find k so that f is continuous. $K = 3$

① $2 + K = 4 + 1$
$K = 3$

30. If $f(x) = \begin{cases} ax - 2, & x \leq 3 \\ x^2 - b, & x > 3 \end{cases}$, find a and b so that f(x) is continuous and differentiable.

① $3a - 2 = 9 - b$

② Der Der at x=3
$a = 2(3)$
$\boxed{a = 6}$

$3(6) - 2 = 9 - b$
$16 - 9 = -b$
$\boxed{-7 = b}$

31. Given that f and g are differentiable functions (-∞, ∞), g(x) <0 for all x's, and f(0)=2,

Neg

if $h(x) = \frac{f(x)}{g(x)}$ and $h'(x) = \frac{-f(x)g'(x)}{[g(x)]^2}$ what can you conclude about f?

$f(x) = 2$

$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

so $g(x) f'(x) = 0$ so either
$g(x) = 0$ or $f'(x) = 0$
can't happen — must be true
$f(x) = $ constant

32. Find the instantaneous rate of change at t = 3 for $h(t)=\dfrac{t^3-3}{t-2}$.

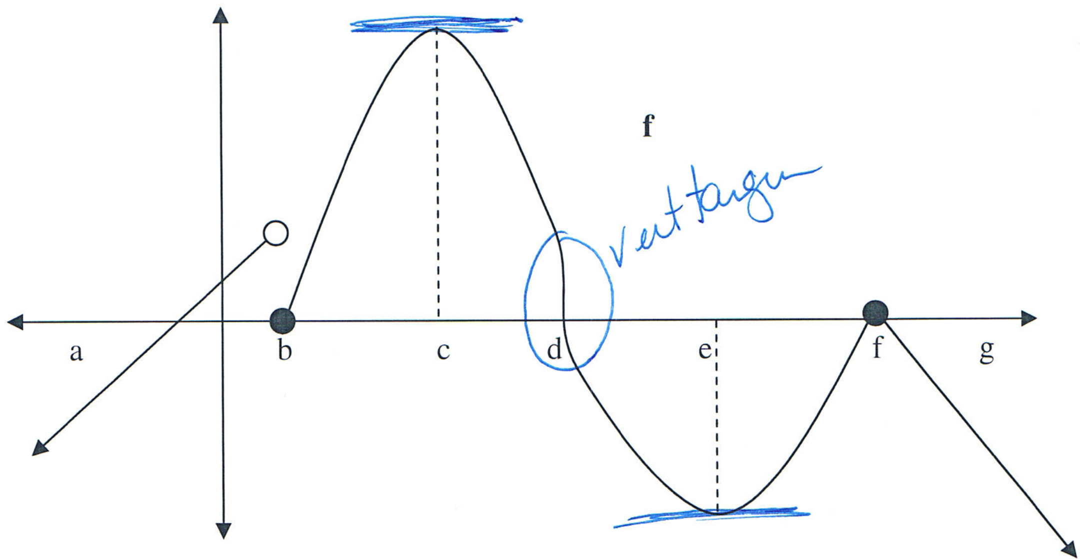
(A calculator is allowed on this problem.)

$h'(t)=\dfrac{(t-2)(3t^2)-(t^3-3)(1)}{(t-2)^2}$ $\quad h'(3)=\dfrac{(1)(27)-24(1)}{(1)}$

$27-24$

$\underline{\quad 3 \quad}$

33. The graph of f has horizontal tangents at c and e, and a vertical tangent at d as indicated on the graph below.



a) On what intervals (-∞, ∞) is f continuous?

$(-\infty,b)\cup(b,+\infty)$

b) On what intervals (a, g) is f differentiable?

$(-\infty,b)\cup(b,d)\cup(d,f)\cup(f,+\infty)$

c) On what intervals is f '(x) < 0? f(x) is decreasing

$(c,d)\cup(d,e)\cup(f,+\infty)$

For questions 34-35 use the following data: $f(2)=3$, $f'(2)=-1$, g(2)=5, g(1)=2, g'(2)=8, $g'(1)=4$.

34. Find h '(2) if h(x) = f(x) g(x). Product

$h'(x)=f(x)g'(x)+g(x)f'(x)$

$h'(2)=f(2)g'(2)+g(2)f'(2)$

$=3(8)+(5)(-1)$

$24-5$

$\boxed{19}$

35. Find t '(1) if t(x)= f(g(x)) Chain

$t'(x)=f'(g(x))\,g'(x)$

$t'(1)=f'(g(1))\,g'(1)$

$=f'(2)\,4$

$=(-1)4 =$ $\boxed{-4}$

36. Find a local linear approximation for f(x) = tan x at x = π and use it to approximate f(3).

① Point

$f(\pi) = \tan \pi$

$= 0$

$(\pi, 0)$

② Slope

$f'(x) = \sec^2(\cancel{x})$

$f'(\pi) = \sec^2(\pi)$

$= \dfrac{1}{(\cos \pi)^2} = \dfrac{1}{+1}$

$f'(\pi) = +1$

③ Eq of tan line

$y - 0 = 1(x - \pi)$

$f(3) \approx 1(3 - \pi)$

$\approx 3 - \pi$

37. Find a local linear approximation for g(x) = ln (3x) at x = e and use it to approximate g(2).

① Point

$(e, \ln(3e))$

$\downarrow$

$\ln 3 + \ln e$

$\ln 3 + 1$

② Slope

$f'(x) = \dfrac{1}{\cancel{3}x} \cdot \cancel{3}$

$f'(e) = \dfrac{1}{e}$

③ Eq of tan line

$y - \ln(3e) = \frac{1}{e}(x - e)$

$y = \frac{1}{e}(x - e) + \ln(3e)$

$g(2) \approx \frac{1}{e}(2 - e) + \ln(3e)$

$\downarrow$

$\frac{2}{e} - 1 + \ln 3 + \ln e$

$\frac{2}{e} - 1 + \ln 3 + 1$

$\boxed{\frac{2}{e} + \ln 3}$