Name______________________________ Class__________ Date______________

AP Calculus ~ Semester I Review
Part 4
Graphing Concepts

1. Given: f is continuous $a \le x \le b$ and differentiable $a < x < b$

a) f '(c)= $\frac{f(b)-f(a)}{b-a}$

b) Under what circumstances must there be an extrema on $a < x < b$? *(Justify.)*

$f'(x)=0$ or und and changes sign around x-value.

c) Must there be an absolute maximum or minimum on $a \le x \le b$? *(Justify.)*

Yes - It will either be $f(a)$, $f(b)$ or $f'(c)=0$ $f(c)$

d) Under what conditions must there be a c, where $a < c < b$, such that f '(c) = 0?

$f(b)=f(a)$

2. Find an equation in slope intercept form for the line tangent to the graph of **y = x – sin x** at the point $(-\pi,-\pi)$.

$y'=1-\cos x$

$y'(-\pi)=1-\cos(-\pi)$

$=1-(-1)$

$=2$

$y+\pi=2(x+\pi)$

$y=2x+2\pi-\pi$

$y=2x+\pi$

3. Find the x coordinate of the point of inflection for $y=\frac{1}{4}x^3+6x^2+32$.

$y'=\frac{3}{4}x^2+12x$

$y''=\frac{3}{2}x+12=0$

$x=-12\cdot\frac{2}{3}=-8$

$\boxed{x=-8}$

4. If f "(x) = x (x + 1)^3 (x – 2)^2 then the graph of f has inflection points at x = ?

$\boxed{x=0}$

$x+1=0$

$\boxed{x=-1}$

$\cancel{(x-2)^2}$ will not change signs

5. On what intervals is the function $f(x) = x^3 + 2x^2 - 4x$ increasing?

$f'(x) = 3x^2 + 4x - 4 = 0$

$(3x-2)(x+2) = 0$

$x = \frac{2}{3}$ $x = -2$

f'(x) Inc + | Dec − | Inc +

NLA −3 −2 0 $\frac{2}{3}$ 1

$(-\infty, -2) \cup (\frac{2}{3}, +\infty)$

6. On what intervals is the function $f(x) = -x^4 - x^2 + 4$ concave down?

$f'(x) = -4x^3 - 2x$

$f''(x) = -12x^2 - 2 = 0$

$x^2 = \frac{-1}{6}$

$(-\infty, +\infty)$

Always neg

7. Find all local extrema by the First Derivative test for
f(x) = x^3- 27x.

$f'(x) = 3x^2 - 27$

$x^2 = 9$

$x = \pm 3$

f'(x) Inc + | Dec − | Inc +

NLA −4 −3 0 3 4

x = −3 f'(x) changes from + to −
rel max

x = 3
rel min f'(x) changes from − to +

8. Find all local extrema by the Second Derivative test for
f(x) = x^3+4x^2+4x-16.

$f'(x) = 3x^2 + 8x + 4 = 0$ $(3x+2)(x+2)$ $x = \frac{-2}{3}$ $x = -2$

$x = -2$ $f'(-2) = 0$ $f''(-2) < 0$ there is a rel max

$f'(-\frac{2}{3}) = 0$ $f''(-\frac{2}{3}) > 0$ there is a rel min

$f''(x) = 6x + 8$

CD − | CU +

−2 $-\frac{4}{3}$

9. On what intervals is the function $f(x) = -x^4 - x^2 + 4$ increasing?
even

f'(x) + | −

NLA −1 0

0

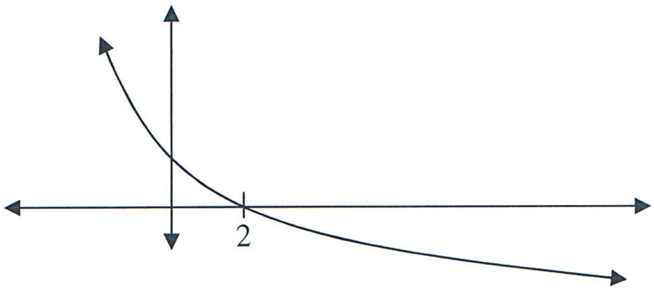
$f'(x) = -4x^3 - 2x = 0$ $-2x(2x^2+1) = 0$

$-2x = 0$ $x^2 = -\frac{1}{2}$

$x = 0$ No Sol

Increasing $(-\infty, 0)$

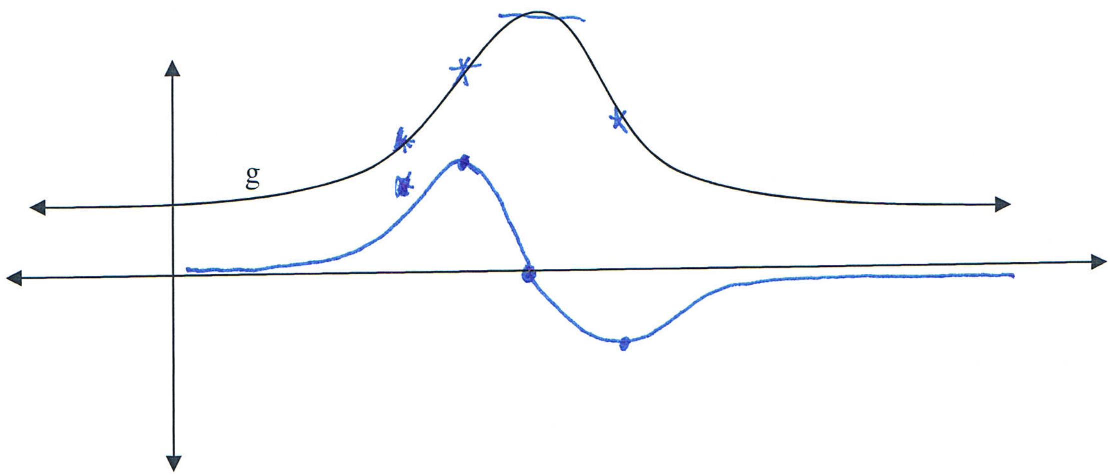
10.



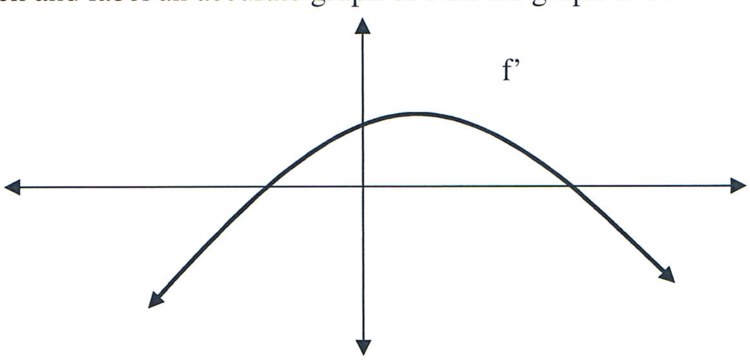
$f(2)=0$
$f'(2)= -$ Dec
$f''(2)= +$ CU

Arrange f(2), f '(2), and f "(2) in order from least to greatest. $f'(2)$ < $f(2)$ < $f''(2)$

11. Sketch and label an accurate graph of g 'on the graph of g.



12. Sketch and label an accurate graph of f on the graph of f'.



13. The function f, is continuous [-1, 3] and has values given in the table below.

x	-1	1	3
f(x)	-2	k	-4

What values of k make it possible that f(x) = 1 would have at least two solutions [-1, 3]?