Name_____ Class_____ Date_____

AP Calculus ~ Semester I Review
Part 3

All problems are to be worked without a calculator unless otherwise noted.

Velocity and Acceleration

1. A particle moves along the x-axis so that its position at time t, is given by $x(t) = t^2 - 8t + 7$.
For what values of t is the velocity zero?

$x'(t) = 2t - 8 = 0$

$t = 4$

2. Given $v(t) = -t^3 + 4t^2 + 3t - 5$.

a) Find the maximum velocity $0 \le t \le 4$

$v'(t)$ changes from + to neg

$v'(t) = -3t^2 + 8t + 3 = 0$

$f(0) = -5$

$f(4) = 7$

$f(3) = 13$

$f(-\frac{1}{3})$ Not on interval

$3t^2 - 8t - 3 = 0 \quad (3t+1)(t-3) = 0$

$t = -\frac{1}{3} \quad 3t+1=0 \quad t-3=0 \quad t=3$

b) Find the maximum acceleration on the interval, $0 \le t \le 4$.

$a'(t) = 0$ changes from + to neg

$a(t) = -3t^2 + 8t + 3$

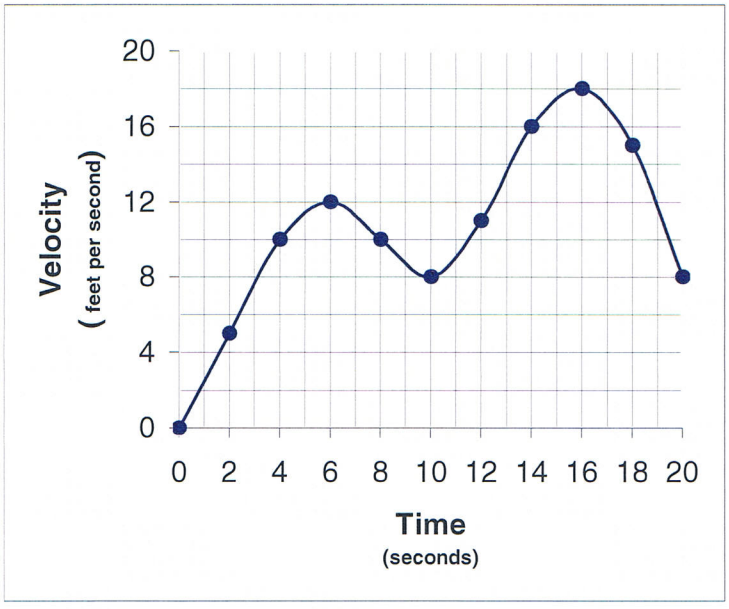
$a'(t) = -6t + 8 = 0 \quad t = \frac{4}{3}$

$a(0) = 3$

$a(4) = -77$

$a(\frac{4}{3}) = -13$

3. Use the graph and the table below to answer the following:
(Usage of a calculator is allowed on this problem.)



t (seconds)	v(t) (ft per second)
0	0
2	5
4	10
6	12
8	10
10	8
12	11
14	16
16	18
18	15
20	8

a) During what intervals is acceleration negative? *(Justify.)*

$v'(t) < 0$ slopes of $v(t)$ are negative

$(6,10)$ $(16,20)$

b) Find the average acceleration in ft/s^2 for $0 \le t \le 12$.

(Show your computation and justify your answer.)

$$\frac{v(12)-v(0)}{12-0} = \frac{11-0}{12} = \frac{11}{12}\ ft/s^2$$

c) Approximate the acceleration in ft/s^2 at t = 8.

(Show your computation and justify your answer.)

$$\frac{v(10)-v(6)}{10-6} = \frac{8-12}{4} = \frac{-4}{4} = -1\ ft/s^2$$

For problems 4-6, use calculus to answer each question. Show all work on separate sheet of paper. A calculator may be necessary.

4. A particle moves along the x-axis so that its position is given by $s(t) = 3 \cos t$ for $0 \leq t < 6.28$.

a) Find the position of the particle at 3 seconds. $s(3) = 3\cos(3) = -2.970$

b) List intervals on which the particle is to the left of the origin.
$s(t) < 0 \quad \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

c) Find a function that describes the velocity of the particle.
$s'(t) = v(t) = -3\sin t$

d) Find the velocity of the particle at 3 seconds.
$v(3) = -3\sin(3) = -.423$

e) List intervals on which the function is moving to the right.
$v(t) > 0 \quad (\pi, 2\pi)$

f) What is the average velocity of the particle between t = 1 and t = 4? $\frac{s(4) - s(1)}{4-1} = -1.194$ u/s

g) At what time is the instantaneous velocity of the particle equal to this average? $s'(t) = \frac{s(4) - s(1)}{4-1}$
$t = .409 \quad t = 2.732$

h) Find a function that describes the acceleration of the particle. $-3\sin(t) = -1.194$
$s''(t) = v'(t) = a(t) = -3\cos t$

i) Find the acceleration of the particle at 3 seconds.
$a(3) = -3\cos 3 = 2.970$

j) List intervals on which the particle is decelerating. $\left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$
$v(t)$ & $a(t)$ opp signs (graph both)

k) Summarize the particles position and movement at t = 3.
Left of origin, decelerating because the particle is getting ready to turn around

5. A particle moves along the x-axis so that its position is given by $s(t) = x^3 - x^2 - 6x$ in cm per second for $0 \leq t < 15$ seconds.

a) Find the position of the particle at 12 seconds.

b) List intervals on which the particle is to the right of the origin.

c) Find a function that describes the velocity of the particle.

d) Find the velocity of the particle at 12 seconds.

e) List intervals on which the function is moving to the left.

f) Find a function that describes the acceleration of the particle.

g) Find the acceleration of the particle at 12 seconds.

h) List intervals on which the particle is decelerating.

i) Summarize the particles position and movement at t = 12.

6. An object is thrown vertically upward at a rate of 5 ft per second, from the top of a 200 ft tower.

a) Write a function that describes its position. $s(t) = 200 + 5t - 16t^2$

b) What is the average velocity of the function on [1, 4]?

c) Find the maximum height and the time at which it occurs.

d) At what time will the object strike the ground?

e) Find a function that describes the velocity of the object.

f) At what time, on the closed interval of the problem situation will the object reach its maximum velocity? What is its maximum velocity?

g) What initial velocity would be necessary for the object to reach a maximum height of 500 feet?