AREA BETWEEN TWO CURVES

1. Find the area between $y = -x(x+2)$ and $y = -5x(x+2)$.

2. Find the area between $y = 4x - x^2$ and $y = x$ from $x = 0$ to $x = 4$. Check answer with TI-89.

AREA BETWEEN TWO CURVES (CONTINUED)

3. Find the area between $y^2 = -x + 3$ and $y = -2x$.

4. Find the line $x = h$ that divides the first quadrant region bounded by the two curves into two regions of equal area. $y = e^{x-1}$ and $y = 2\sin x$.

AREA USING HORIZONTAL RECTANGLES

5. Find the area of the region bounded by $x = y^2 - 7y + 6$ and the *y*-axis.

6. Find the area of the region bounded by $y^2 = -x + 3$ and $y = -2x$. Use TI-89 to evaluate integral.

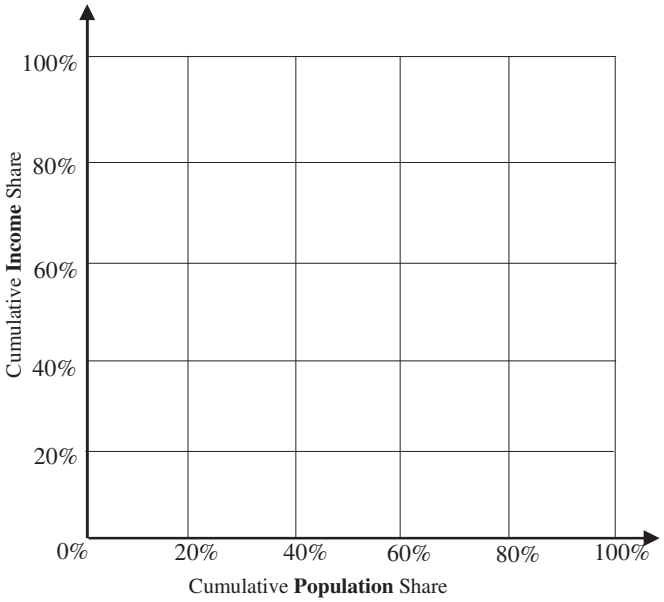
7. Find the area of the region bounded by $x = y^2 - 9$ and $x = \ln y + 2$.

8. Find the area of the region bounded by $y = x^3 + 2x^2 - 4x$ and $y = -x$.

GINI INDEX

9. Consider a population where every worker earns the same income.

   a. Make a graph of cumulative population share versus cumulative income share, distributing the population into quintiles. List the ordered pairs.



   b. Note that the graph is a simple diagonal line. It is called the *Line of Income Equality* or the *Egalitarian Line*.

10. Now consider these statistics for the U.S. population in 1988. Assume that incomes have been ordered from least to greatest.

    lowest quintile of the population had 4.6% of the income.

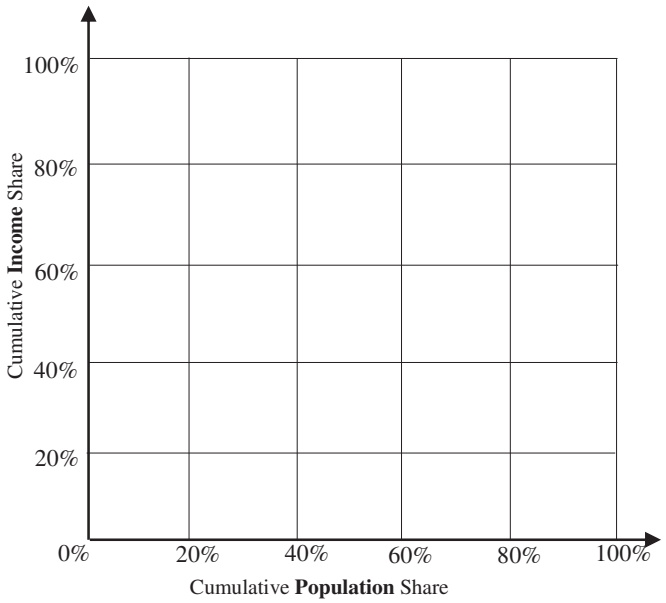
    2nd quintile of the population had 10.7% of the income.

    3rd quintile of the population had 16.7% of the income.

    4th quintile of the population had 24.0% of the income.

    highest quintile of the population had 44.0% of the income.

    a. Use these values to plot cumulative population share versus cumulative income share. List the ordered pairs.



b. Find a best-fit curve for the data.

c. This curve is called the *Lorenz Curve*.

d. Calculate the area between the Lorenz curve and the Egalitarian Line

e. The Gini Index is a relative index of income inequity. $0 \le \text{Gini Index} \le 1$. The larger the value of the Gini Index, the more inequitable the income distribution.

11. Lab Demonstration: Perform the calculations on a StudyWorks worksheet with the data from 1929.

lowest quintile of the population had 0.03% of the income.

2nd quintile of the population had 12.47% of the income.

3rd quintile of the population had 13.8% of the income.

4th quintile of the population had 19.3% of the income.

highest quintile of the population had 54.4% of the income.

BUILDING AN INTEGRAL

12. Suppose that someone has done a population study in Pennsylvania and they have developed some formulas describing the population density in and around Newtown Square. We are interested in the formula $D(x)$ that describes the population density along the 20-mile stretch of Route 3 that extends from Philadelphia west to West Chester. For example, the value $D(6)$ tells how many people there are per square mile if you are 6 miles west of Philadelphia. We are going to use this formula to calculate the total population along a 2 mile wide path following Route 3 from Philadelphia to West Chester.

13. Suppose someone has a well-used dartboard and they have studied the distribution of holes on the dartboard. Their findings are that the holes are clustered much more closely at the center of the board and more sparsely towards the edges. Indeed the density of holes can be described as a function $D(r)$ where r is the distance in centimeters from the center of the board. The board has a radius of 40 cm. We want to calculate the total number of holes on the board.

VOLUMES OF SOLIDS OF KNOWN CROSS-SECTION

14. The base of a solid is a circle of radius 1. Parallel cross sections cut perpendicular to the base form rectangles whose height is 2. Find the volume of the solid.

15. The base of a solid is an equilateral triangle of side 2. Parallel cross sections cut perpendicular to the base of the triangle form a square. Find the volume of the solid.

16. The base of a solid is a circle of radius 2. Parallel cross sections cut perpendicular to the base are equilateral triangles. Find the volume of the solid.

17. The base of a solid is a region bounded by the curve $y = 4 - x^2$ and the x-axis. Parallel cross sections cut perpendicular to the base and perpendicular to the x-axis are semi-ellipses of height 6. Find the volume.

VOLUMES OF SOLIDS OF REVOLUTION – DISC METHOD

18. Given a region bounded by the curve $y = x^2$, $y = 0$, and $x = 2$. Find the volume of the solid formed by revolving the region about the *x*-axis.

19. Given a region bounded by the curve $y = x^3$, $y = 1$, and $x = 0$. Find the volume of the solid formed by revolving the region about the *y*-axis.

20. Given a region in the *first* quadrant bounded by the curve $y = x^2$ and $y = \sqrt{x}$. Find the volume of the solid formed by revolving the region about the *x*-axis.

21. Find the volume of the solid generated by revolving the region bounded by $y = e^x$, the *y*-axis, and $y = e$ about the *x*-axis.

REVOLVING ABOUT A LINE OTHER THAN AN AXIS

22. Given a region bounded by the curve $y = x^3$, $y = 0$, and $x = 1$. Find the volume of the solid formed by revolving the region about the line $x = 1$.

23. Given a region bounded by the curve $y = x(x-2)^2$ and $y = 0$. Find the volume of the solid formed by revolving the region about the line $y = 2$.

24. Given a region in the second quadrant bounded by the curve $x = y^3 - y$ and $x = 0$. Find the volume of the solid formed by revolving the region about the line $x = -2$.

25. Given a region bounded by the curve $x - y = -1$, $y = 0$, and $x = 0$. Find the volume of the solid formed by revolving the region about the line $x = 1$.

WORK DONE TO LIFT A LIQUID

26. If a 50 pound weight is lifted 6 feet, find the work done.

27. A tank having the shape of a right circular cylinder of altitude 8 feet and radius of base 5 feet is full of water. Find the work done to pump the water over the top edge.

28. An inverted right circular cone with radius 3 feet and height 10 feet is full of water. Find the work required to pump all of the water out of the top of the tank and into a second tank whose top is 5 feet above the top of the cone.

29. A trough 6 feet long is half full of water. Its cross section is in the shape of a semi-circle with diameter 2 ft at the top. If the trough develops a leak at the bottom, how much work is done as the trough empties?