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| 1 | Limits at Infinity:<br><br>To find $\lim_{x\to\pm\infty} f(x)$ think<br>Top Heavy $\Rightarrow$ limit is $\pm\infty$<br>Bottom Heavy $\Rightarrow$ limit is 0<br>Equal $\Rightarrow$ limit is ratio of coefficients |
| 2 | Limits with Infinity (at vertical asymptotes):<br><br>When finding a one-sided limit at a vertical asymptote,<br>the answer is either $\pm\infty$. |
| 3 | *Justifying* that a function is continuous at a point:<br><br>$f$ is continuous at $c$ iff:<br>1. $f(c)$ is defined<br>2. $\lim_{x\to c} f(x)$ exists<br>3. $f(c) = \lim_{x\to c} f(x)$ |
| 4 | Definition of the Derivative:<br><br>$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$<br><br>$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ (Alternate form for a derivative at a given value.) |
| 5 | *Justifying* that a derivative exists at a point, $c$:<br><br>Show algebraically that $\lim_{x\to c^-} f'(x) = \lim_{x\to c^+} f'(x)$. |
| 6 | *Average* Rate of Change of $f$ on [a, b]:<br><br>a.r.c. = $\frac{f(b)-f(a)}{b-a}$ (algebra slope of $\frac{\Delta y}{\Delta x}$) (slope of secant line) |
| 7 | *Instantaneous* Rate of Change of $f$ at $a$:<br><br>$f'(a)$ (derivative at the given value) (slope of tangent line) |

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| 8 | Power Rule: $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$ |
| 9 | Common Derivatives to Remember: $\frac{d}{dx}\left[\frac{1}{x}\right]=\frac{-1}{x^2}$ $\frac{d}{dx}\left[\sqrt{x}\right]=\frac{1}{2\sqrt{x}}$ |
| 10 | Trig Function Derivatives: $\frac{d}{dx}\left[\sin x\right]=\cos x$ $\frac{d}{dx}\left[\cos x\right]=-\sin x$ $\frac{d}{dx}\left[\tan x\right]=\sec^2 x$ $\frac{d}{dx}\left[\cot x\right]=-\csc^2 x$ $\frac{d}{dx}\left[\sec x\right]=\sec x\tan x$ $\frac{d}{dx}\left[\csc x\right]=-\csc x\cot x$ |
| 11 | Derivatives of Inverse Trig Functions: $\frac{d}{dx}[\arcsin x]=\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}[\arccos x]=\frac{-1}{\sqrt{1-x^2}}$ $\frac{d}{dx}[\arctan x]=\frac{1}{1+x^2}$ $\frac{d}{dx}[arc\cot x]=\frac{-1}{1+x^2}$ $\frac{d}{dx}[arc\sec x]=\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ $\frac{d}{dx}[arc\csc x]=\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$ |
| 12 | Derivatives of Exponential and Logarithmic Functions: $\frac{d}{dx}[\ln x]=\frac{1}{x},\ \ x>0$ $\frac{d}{dx}[\log_a x]=\frac{1}{x\ln a}$ $\frac{d}{dx}[e^x]=e^x$ $\frac{d}{dx}[a^x]=a^x\ln a$ |
| 13 | *<u>Justifications</u>* for horizontal tangent lines: $f(x)$ has horizontal tangents when $\frac{dy}{dx}=0$. |

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| 14 | Chain Rule: <br><br> $\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$ $\qquad\qquad$ $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \bullet g'(x)$ |
| 15 | Product Rule: <br> $\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$ |
| 16 | Quotient Rule: <br><br> $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$ |
| 17 | Derivatives of Inverse Functions: <br><br> The derivative of an inverse function is the reciprocal of the derivative of the original function at the "matching" point. <br> If (a, b) is on $f(x)$, then (b, a) is on $f^{-1}(x)$ and $(f^{-1})'(b) = \frac{1}{f'(a)}$. |
| 18 | *Justifications* for horizontal tangent lines: <br><br> $f(x)$ has vertical tangents when $\frac{dy}{dx}$ is undefined. |
| 19 | *Justifications* for Particle Motion: <br><br> Particle is moving right/up because $v(t) > 0$ (positive). <br> Particle is moving left/down because $v(t) < 0$ (negative). <br> Particle is speeding up (\|velocity\| is getting bigger) because $v(t)$ and $a(t)$ have same sign. <br> Particle is slowing down (\|velocity\| is getting smaller) because $v(t)$ and $a(t)$ have different signs. |
| 20 | Intermediate Value Theorem: <br><br> If $f$ is continuous on [a, b] and $k$ is any number between $f(a)$ and $f(b)$, then there is at least one number $c$ between $a$ and $b$ such that $f(c) = k$. |
| 21 | Extreme Value Theorem: <br><br> If $f$ is continuous on the closed interval [a, b], then $f$ has both a minimum and a maximum on the closed interval [a, b]. |

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| 22 | *Justification* for an Absolute Extrema.<br><br>1. Find critical numbers.<br>2. Identify endpoints.<br>3. Find $f(\text{critical numbers})$ and $f(\text{endpoints})$.<br>4. Determine absolute max/min values by comparing the y-values. State in a sentence. |
| 23 | Mean Value Theorem:<br><br>If $f$ is continuous on [a, b] and differentiable on (a, b) then there exists a number $c$ on (a, b) such that $f'(c)=\frac{f(b)-f(a)}{b-a}$.<br><br>(Calculus slope = Algebra Slope) |
| 24 | Rolle's Theorem:<br><br>If $f$ is continuous on [a, b] and differentiable on (a, b) and if $f(a)=f(b)$, then there exists a number $c$ on (a, b) such that $f'(c)=0$. |
| 25 | *Justification* for a Critical Number:<br><br>$x=c$ is a critical number because $f'(x)=0$ or $f'(x)$ is undefined. |
| 26 | *Justification* for Increasing/Decreasing Intervals:<br><br>Inc: $f(x)$ is increasing on [_____, _____] b/c $f'(x)>0$.<br><br>Dec: $f(x)$ is decreasing on [_____, _____] b/c $f'(x)<0$. |
| 27 | *Justification* for a Relative Max/Min Using 1st Derivative Test:<br><br>Local Max: $f'(x)$ changes from + to -.<br><br>Local Min: $f'(x)$ changes from - to +. |
| 28 | *Justification* for Relative Max/Min Using 2nd Derivative Test:<br><br>Local Max: $f'(c)=0$ (or und) and $f''(x)<0$.<br><br>Local Min: $f'(c)=0$ (or und) and $f''(x)>0$. |

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| 29 | *Justification* for a Point of Inflection:<br><br>Using 2nd derivative: $f''(x)=0$ (or dne) AND $f''(x)$ changes sign.<br><br>Using 1st derivative: $f''(x)=0$ (or dne) AND slope of $f'(x)$ changes sign. |
| 30 | *Justification* for Concave Up/Concave Down:<br><br>Concave Up: $f(x)$ is concave up on (_____, _____) because $f''(x)>0$.<br><br>Concave Down: $f(x)$ is concave down on (_____, _____) because $f''(x)<0$. |
| 31 | *Justifications* for linear approximation estimates:<br><br>A linear approximation (tangent line) is an *overestimate* if the curve is concave down.<br>A linear approximation (tangent line) is an *underestimate* if the curve is concave up. |
| 32 | Integration Rules:<br><br>$\int x^n dx = \frac{1}{n+1}x^{n+1}+C$    $\int \cos x dx = \sin x + C$<br>$\int \cos(kx)dx = \frac{1}{k}\sin(kx)+C$    $\int \sin x dx = -\cos x + C$<br>$\int \sin(kx)dx = -\frac{1}{k}\cos(kx)+C$    $\int \sec^2 x dx = \tan x + C$<br>$\int \csc^2 x dx = -\cot x + C$    $\int \sec x \tan x dx = \sec x + C$<br>$\int \csc x \cot x dx = -\csc x + C$    $\int \frac{1}{x}dx = \ln\lvert x\rvert + C$<br>$\int \tan x dx = -\ln\lvert\cos x\rvert + C$    $\int e^x dx = e^x + C$<br>$\int e^{kx}dx = \frac{1}{k}e^{kx}+C$    $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+C$ |

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| 33 | *Justifications* for Reimann Sums:<br><br>Left-Riemann Sums:<br>The sum is an *overestimate* if the curve is decreasing.<br>The sum is an *underestimate* if the curve is increasing.<br><br>Right-Riemann Sums:<br>The sum is an *overestimate* if the curve is increasing.<br>The sum is an *underestimate* if the curve is decreasing. |
| 34 | First Fundamental Theorem of Calculus:<br><br>$\int_a^b f'(x)dx = f(b) - f(a)$<br>(Finds the signed area between a curve and the x-axis) |
| 35 | Properties of Integrals:<br><br>$\int_a^b f(x) + g(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$<br>$\int_a^b f(x) - g(x)dx = \int_a^b f(x)dx - \int_a^b g(x)dx$<br>$\int_a^b cf(x)dx = c\int_a^b f(x)dx$<br>$\int_a^b f(x)dx = -\int_b^a f(x)dx$<br>$\int_a^a f(x)dx = 0$ |
| 36 | Average Value of a Function:<br><br>$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$ |
| 37 | Second Fundamental Theorem of Calculus:<br><br>$\frac{d}{dx}\int_a^x f(t)dt = f(x)$ $\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x)) \bullet g'(x)$ |

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| 38 | "Net Change" Theorem:<br><br>$\int_a^b f(x)dx$ represents the "net change" in the function $f$ from time a to b. |
| 39 | Finding Total Amount:<br><br>$f(b) = f(a) + \int_a^b f'(x)dx$ (want = have + integral) |
| 40 | Steps for Solving Differential Equations:<br><br>"Find a solution (or solve) the separable differentiable equation…"<br>1. Separate the variables<br>2. Integrate each side<br>3. Make sure to put C on side with independent variable (normally x)<br>4. Plug in initial condition and solve for C (if given)<br>5. Solve for the dependent variable (normally y) |
| 41 | Exponential Growth and Decay:<br><br>"The rate of change of a quantity is directly proportional to that quantity"<br>Gives the differential equation: $\frac{dy}{dt} = ky$<br>Which can be solved to yield: $y = Ce^{kt}$ |

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| 42 | Particle Motion Formulas:<br><br>Velocity: $v(t) = s'(t)$<br>Acceleration: $a(t) = v'(t) = s''(t)$<br>Speed: speed $= \lvert v(t) \rvert$<br>Average Velocity: (given $s(t)$) $\dfrac{s(b)-s(a)}{b-a}$<br>(given $v(t)$) $\dfrac{1}{b-a}\int_a^b v(t)dt$<br>Average Acceleration: (given $v(t)$) $\dfrac{v(b)-v(a)}{b-a}$<br>(given $a(t)$) $\dfrac{1}{b-a}\int_a^b a(t)dt$<br>Displacement: $\int_a^b v(t)dt$<br>Total Distance: $\int_a^b \lvert v(t) \rvert dt$<br>Position at b: $s(b) = s(a) + \int_a^b v(t)dt$ |
| 43 | Areas in a Plane:<br><br>Perpendicular to x-axis: $\int_a^b [f(x)-g(x)]dx$<br>$f(x)$ is top curve, $g(x)$ is bottom curve, a and b are x-coordinates of point of intersection<br>Perpendicular to y-axis: $\int_a^b [f(y)-g(y)]dy$<br>$f(y)$ is right curve, $g(y)$ is left curve, a and b are y-coordinates of point of intersection |
| 44 | Steps to Finding Volume:<br><br>Volume = $\int Area$<br>1. decide on whether it's a dx or dy<br>2. find a formula for the area in terms of x or y<br>3. find the limits (making sure they match x or y)<br>4. integrate and evaluate |

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| 45 | Volumes Around a Horizontal Axis of Rotation or Perpendicular to x-axis:<br><br>Disc: $V=\int_a^b \pi r^2 dx$ a and b are x-coordinates<br>Washer: $V=\int_a^b \left[\pi R^2-\pi r^2\right]dx$ a and b are x-coordinates<br>Slab (Cross Section): $V=\int_a^b A(x)dx$ $A(x)$ is the area formula for the cross section |
| 46 | Volumes Around a Vertical Axis of Rotation or Perpendicular to y-axis:<br><br>Disc: $V=\int_a^b \pi r^2 dy$ a and b are y-coordinates<br>Washer: $V=\int_a^b \left[\pi R^2-\pi r^2\right]dy$ a and b are y-coordinates<br>Slab (Cross Section): $V=\int_a^b A(y)dy$ $A(y)$ is the area formula for the cross section |