Sect. 10 The Ellipse

An **ellipse** is the locus of all points in a plane such that the sum of the distances from two given points in the plane, called foci, is constant.

The lengths of the blue and green lines are the same.

Every ellipse has two **axes of symmetry**.

*MAJOR AXIS – the longer one
- it always includes the foci

*MINOR AXIS – the shorter one

The intersection of the two axes is the **center** of the ellipse.

The endpoints of the axes are called **vertices**.

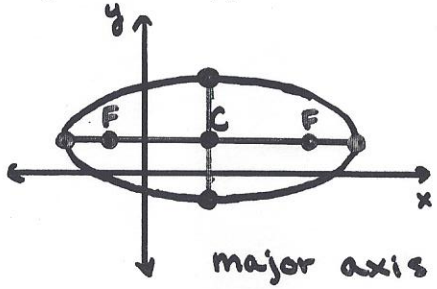
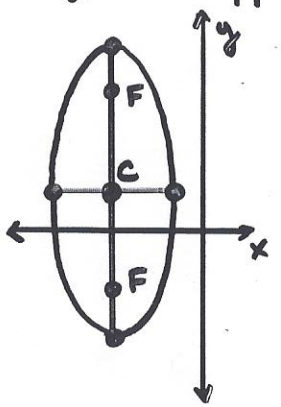
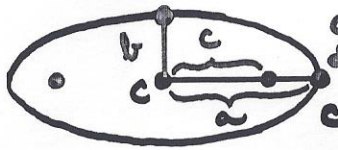
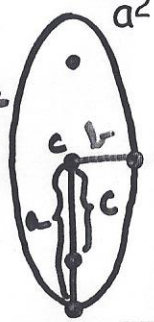
*semi-major axis – from the center to a vertex on the major axis
- its length is a

*semi-minor axis – from the center to a vertex on the minor axis
- its length is b

The distance from the center to either FOCUS is c.

The relationship between a, b, and c is

$a^2 = c^2 + b^2$.

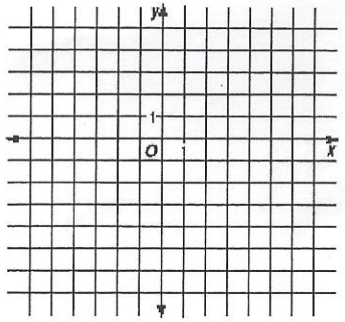
| HORIZONTAL ELLIPSE | VERTICAL ELLIPSE |
|---|---|
| major axis $\|\|$ to x-axis | major axis $\|\|$ to y-axis |
|   |   |
| major axis<br>minor axis<br>• foci<br>• vertices<br>• center | |
| center (h,k) | center (h,k) |
| $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ | $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$ |
|   |   |
| c - distance from C to focus<br>a - distance (major) from C to vertex<br>b - distance (minor) from C to vertex | When finding coordinates of points, a and c always get +/- with the major & b gets +/- with the minor. |

NOTE: a is always greater that b, even if the equation is written in the "wrong" order

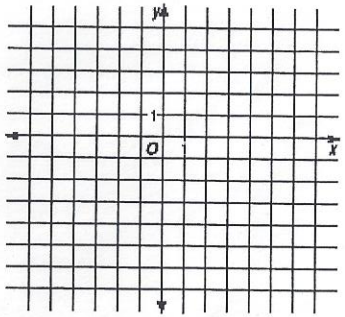
How would an equation of an ellipse in general form differ from the equation of a circle in general form?

**Examples**: Find the coordinates of the center, foci, and the vertices of each ellipse. Then graph each ellipse.

1) $\frac{(y+2)^2}{25}+\frac{(x-3)^2}{16}=1$



2) $4x^2 + 9y^2 - 8x - 54y + 49 = 0$



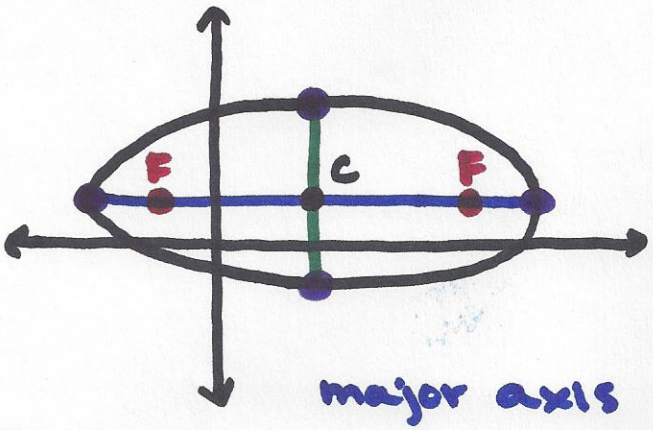
3) Write the equation of the ellipse if the foci are located at (-2,1) and (-2,-7), and the length of the semi-major axis is 5.

The relationship between a, b, and c is

$$a^2 = c^2 + b^2.$$

## HORIZONTAL ELLIPSE

major axis || to x-axis



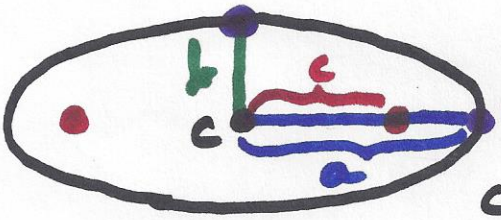

major axis

minor axis

- foci
- center
- vertices

center (h,k)

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



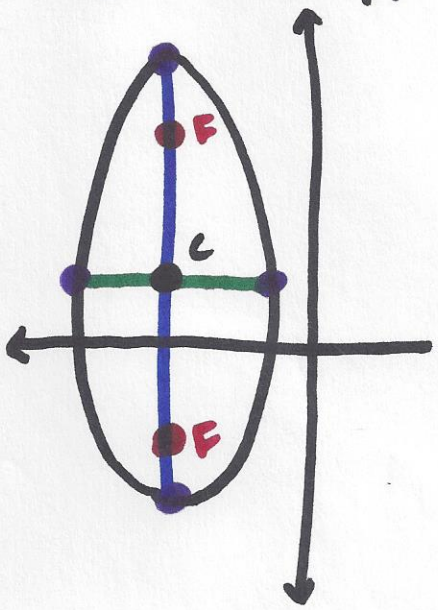

c - distance from c to focus

a - distance (major) from c to vertex

b - distance (minor) from c to vertex

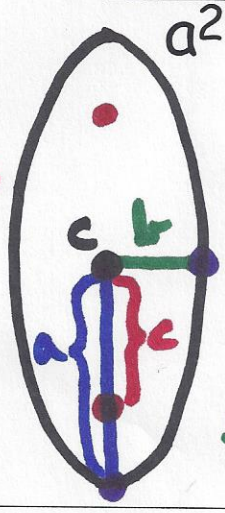
## VERTICAL ELLIPSE

major axis || to y-axis




center (h,k)

$$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$




When finding coordinates of points, a and c always get +/- with the major & b gets +/- with the minor.

**NOTE**: a is always greater that b, even if the equation is written in the "wrong" order