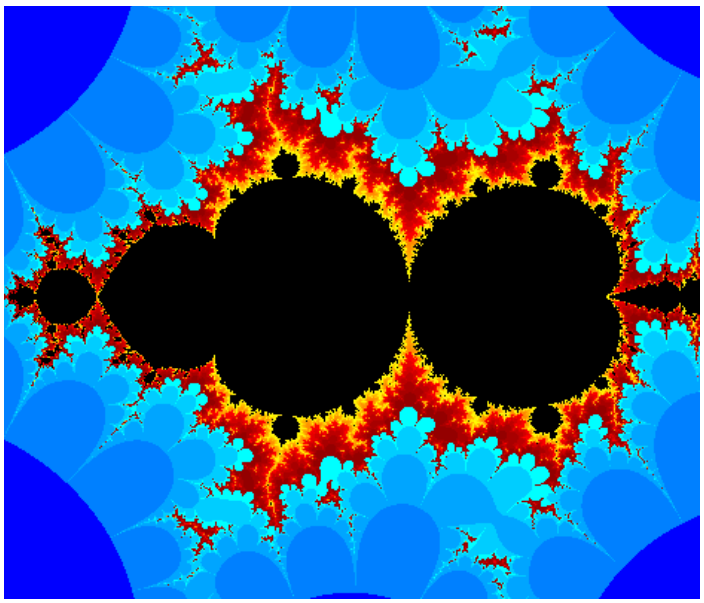
Sinusoidal Applications

A package of 5 activities

Problems dealing with graphing and determining the equations of sinusoidal functions for real world situations



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Sinusoidal Applications

Activity #1 - Temperatures in Florence

From a set of data:

1. Determine the period, amplitude, phase and vertical shifts
2. Determine sine and cosine equations from information in #2
3. Perform a sine regression using the Ti83/84

Activity #2 - Steamboat problem

From given information:

1. Create a sinusoidal wave from given information
2. Give the amplitude, period, phase and vertical shifts
3. Determine an equation to suit the graph
4. Apply the equation

Activity #3 – Fox Population Problem

From given information:

1. Create a sinusoidal wave from given information
2. Give the amplitude, period, phase and vertical shifts
3. Determine an equation to suit the graph
4. Apply the equation

Activity #4 – The Beijing Wheel

From given information:

1. Create a sinusoidal wave from given information and determine some missing information
2. Give the amplitude, period, phase and vertical shifts
3. Determine the speed of the rider
4. Determine an equation to suit the graph

Activity #5 – Hours of Sunlight in Zushi, Japan 2001

From a set of data:

1. Plot the data and create a sinusoidal curve
2. Determine a sine and cosine equation from the graph
3. Perform a sine regression using the Ti83/84 from the data given and compare the results to #2

Activity #1

Temperatures in Florence

Below is the data for average temperature per month for Florence, Italy. Located at *about* 43.80°N 11.30°E. Height *about* 75m / 246 feet above sea level.

Average Temperature

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
°C	5.2	6.6	9.4	13.3	17.9	22.0	24.6	24.0	20.2	15.0	10.0	6.2	14.5
°F	41.4	43.9	48.9	55.9	64.2	71.6	76.3	75.2	68.4	59.0	50.0	43.2	58.1

Source: derived from 884 months between 1832 and 1981

1. Plot the data. The data appears roughly sinusoidal. (use F scale the spread is better)

2. Model this graph with a sinusoidal function. Looking at the data, determine the **period**, **amplitude**, and the **vertical** and **horizontal** translations. Using those values determine a cosine and sine equation for this curve sinusoidal equation and test your model by graphing it on top of the data. (It won't be exact, but it should at least hit some of the points and be relatively close to the others.)

3. Compare this with the equation found using the sinusoidal regression with the Ti83/84.

Activity #2

Steamboat Problem

Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When his stopwatch read 4 s, the point was at its highest, 16 ft above the water's surface. The wheel's diameter was 18 ft, and it completed a revolution every 10 s.

1. Sketch a graph of the sinusoidal curve.

2. Find the amplitude, phase shift, period, and translations.

3. Write the equation of the graph.

4. Predict the height or depth of the point from the surface of the water when Mark's stopwatch read:

a) 5 s

b) 17 s

c) 0 s

d) 4 s

Activity #3

Fox Population Problem

Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time t = 0 years. A minimum number, 200 foxes, occurred when t = 2.9 years. The next maximum, 800 foxes, occurred at t = 5.1 years.

1. Sketch a graph of this sinusoidal curve.

2. Find the amplitude, phase shift, period changes, and translations.

3. Write an equation of the graph expressing the number of foxes as a function of time.

4. Predict the population at the following times:

a) t = 7 years

b) t = 0 years

c) t = 9.5 years

d) t = 2 years.

Activity #4

The Beijing Wheel

The **Beijing Great Wheel** is an observation wheel currently undergoing construction** in Beijing, People's Republic of China. On completion in late 2009, it was to be the highest and largest Ferris wheel in the world. Planned to stand at 208 m (680 ft), significantly higher than both the current record holder, the 165 m (540 ft) Singapore Flyer, and the 185 m (610 ft) Great Berlin Wheel, currently undergoing construction also.

The wheel will be located in eastern Beijing's Chaoyang Park, where Olympic events, such as volleyball, took place at the 2008 Summer Olympics. The wheel will have 48 air conditioned observation capsules that can carry up to 40 passengers. On a clear sunny day, it is expected that people will be able to see the Great Wall of China in the mountains to Beijing's north.

***Note: On 3 May 2010 it was reported that Great Beijing Wheel Co., the company set up to build the wheel, had gone into receivership after breaching the conditions of a loan, with Ferrier Hodgson and Zolfo Cooper appointed as administrative receivers. Construction work has been stopped since then.*

Key information: radius of the wheel = 97m

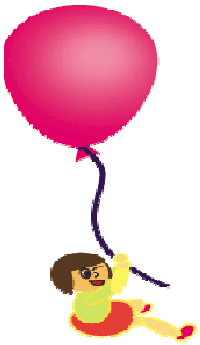
Height at the bottom of the wheel = _____ m

Height at the top of the wheel = 208 m

Period = 20 minutes

1. Fill in the missing information.
2. On graph paper create a graph showing the height of a rider for two revolutions. Label the period, amplitude, and the equation of the axis.
3. Determine the speed of the rider in cm/minute.
4. Determine the sinusoidal equation for the height of the rider

Activity #5



Hours of Sunlight in Zushi, Japan 2001

The table below shows the data for the hours of sunlight for Zushi, Japan

1. Plot the data on graph paper.
2. Determine a sine and cosine equation for the number of hours of sunlight.
3. Place the data into the graphing calculator and do a sine regression. How well does this compare to the equation you determined manually?

City: Zushi, Japan			Latitude: 35 N		Longitude: 135 E	
Date	Day of the Year	Sunrise	Sunrise	Sunset	Sunset	**Hours of Daylight**
01 01	**0**	7:08	7.13	16:59	16.98	**9.85**
01 15	**14**	7:08	7.13	17:11	17.18	**10.05**
01 29	**28**	7:01	7.02	17:25	17.42	**10.40**
02 12	**42**	6:50	6.83	17:39	17.65	**10.82**
02 26	**56**	6:34	6.57	17:53	17.88	**11.31**
03 12	**70**	6:15	6.25	18:05	18.08	**11.83**
03 26	**84**	5:56	5.93	18:16	18.27	**12.34**
04 09	**98**	5:37	5.62	18:27	18.45	**12.83**
04 23	**112**	5:19	5.32	18:39	18.65	**13.33**
05 07	**126**	5:04	5.07	18:50	18.83	**13.76**
05 21	**140**	4:53	4.88	19:01	19.02	**14.14**
06 04	**154**	4:47	4.78	19:10	19.17	**14.39**
06 19	**168**	4:46	4.77	19:16	19.27	**14.50**
07 02	**182**	4:50	4.83	19:18	19.30	**14.47**
07 16	**196**	4:58	4.97	19:14	19.23	**14.26**
07 29	**210**	5:08	5.13	19:05	19.08	**13.95**
08 13	**224**	5:18	5.30	18:51	18.85	**13.55**
08 27	**238**	5:29	5.48	18:34	18.57	**13.09**
09 10	**252**	5:39	5.65	18:15	18.25	**12.60**
09 24	**266**	5:49	5.82	17:55	17.92	**12.10**
10 08	**280**	6:00	6.00	17:35	17.58	**11.52**
10 22	**294**	6:12	6.20	17:17	17.28	**11.08**
11 05	**308**	6:24	6.40	17:02	17.03	**10.63**
11 19	**322**	6:38	6.63	16:52	16.87	**10.24**
12 03	**336**	6:51	6.85	16:48	16.80	**9.95**
12 17	**350**	7:02	7.03	16:50	16.83	**9.80**
12 31	**364**	7:08	7.13	16:58	16.97	**9.84**

Answer Keys

Activity #1 – **Temperatures in Florence**

1. Amplitude ≈ 17.4
   Period = 12 month
   Vertical shift 58.9 up
   Horizontal shift – for cosine 7 right
   For sin 4.5 right
   Cosine equation $y = 17.4\cos\frac{2\pi}{12}(x-7)+58.9$
   Sine equation $y = 17.4\sin\frac{2\pi}{12}(x-4.5)+58.9$

3. Ti83/84 equation $y = 17.4\sin(0.544x-2.328)+58.8$

$y = 17.4\sin\frac{2\pi}{12}(x-4.5)+58.9 \approx y = 17.4\sin(0.524-2.356)+58.9$

Activity #2 - **Steamboat Problem**

2. Amplitude 9
   Period = 10 seconds
   Vertical shift 7 up
   Horizontal shift 4 right(using the cosine function), 2 right (sine)

3. $d = 9\cos\frac{2\pi}{10}(t-4)+7$ or $d = 9\sin\frac{2\pi}{10}(t-2)+7$ ** other answers possible

4. a) at t = 5, d = 14.3 feet (above the water)
   b) at t = 17, d = 4.2 feet
   c) at t = 0, d = -0.28 feet (underwater)
   d) at t = 4, d = 16 feet

Activity #3 - **Fox Population Problem**

1. Amplitude 300
   Period = 4.4 seconds
   Vertical shift 500 up
   Horizontal shift 4 right for a sine, 5.1 right for a cosine

3. $y = 300 \sin\frac{2\pi}{4.4}(t-4)+500$, $y = 300 \cos\frac{2\pi}{4.4}(t-5.1)+500$

** other answers are possible

4. a) t = 7 there are 227 foxes
   b) t = 0 there are 662 foxes
   c) t = 9.5 there are 800 foxes
   d) t = 2 there are 415 foxes

Activity #4 - **The Beijing Wheel**

1. Height at the bottom of the wheel is 14m
2. Period = 20 min, amplitude = 97, axis at 111 m
3. Speed = 3046 cm/sec
4. $h = 97 \sin\frac{2\pi}{20}(t-5)+111$

Activity #5 – **Hours of Sunlight in Zushi, Japan 2001**

2. $h = 2.35 \cos\frac{2\pi}{365}(t-168)+12.15$ and $h = 2.35 \sin\frac{2\pi}{365}(t-80)+12.15$
3. $y = 2.320\sin(0.0168x-1.318)+12.127$ for the regression

   $\frac{2\pi}{365} \approx 0.0172$ and $\frac{2\pi}{365}(80) \approx 1.377$

This would make the hand calculation $h = 2.35 \sin(0.0172t-1.377)+12.15$ which is very close