Geometric Sequences and Series

Note Title 9/13/2010

Objectives: To 1) find terms of a geometric sequence + 2) find the sum of a geometric series

$\div 2$

$r = \frac{4}{8}$ $8, 4, 2, 1, \ldots$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$ $* \frac{1}{2}$ or 0.5

$\frac{2\sqrt{2}}{2}$ $2, 2\sqrt{2}, 4, 4\sqrt{2}, \ldots 8, 8\sqrt{2}, 16, \ldots$ $* \sqrt{2}$

$0.08, 0.4, 2, 10, \ldots$ $\frac{10}{2} = 5$ $* 5$ $50, 250, 1250$

$r-1, -3r+3, 9r-9, \ldots -27r+27, 81r-81, -243r+243$

$* -3$

geometric: repeatedly multiplying by a constant value → common ratio (r)

$$r = \frac{\text{any term}}{\text{previous term}}$$

Ex 1: Determine the common ratio and find the next three terms in each sequence

a) $1, -\frac{1}{2}, \frac{1}{4}, \ldots -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}$

$r = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$

b) $2t-10, -4t+20, 8t-40, \ldots -16t+80, 32t-160$

$r = -2$

$-64t+320$

factor!

$r = \frac{-4t+20}{2t-10} = \frac{-4\cancel{(t-5)}}{2\cancel{(t-5)}} = -2$

$* \sqrt{3}$

Ex2 Find the tenth term: $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ $r = \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$

$a_{10} = \sqrt{3}^1 (\sqrt{3})^9$

$= 3^5 = \boxed{243}$

$\sqrt{3}\sqrt{3}$

3

$a_n = a_1 r^{n-1}$

Ex3 Write a sequence that has two geometric means between 128 and 54.

$\underset{a_1}{\overset{1}{128}}, \ \overset{2}{\underline{96}}, \ \overset{3}{\underline{72}}, \ \underset{a_n}{\overset{4}{54}}$

$n = 4$

$\overset{32}{\cancel{128}}\left(\frac{3}{\cancel{4}}\right) = 96$

$\overset{24}{\cancel{96}}\left(\frac{3}{\cancel{4}}\right) = 72$

$a_n = a_1 r^{n-1}$

$\frac{54}{128} = \frac{128 r^3}{128}$

$\sqrt[3]{r^3} = \sqrt[3]{\frac{27}{64}}$

$r = \frac{3}{4}$

$r = \frac{-48}{16} = -3$

Ex4: Find the sum of the first $\boxed{\text{ten}}$ terms of the geometric series $16 - 48 + 144 - 432 + \dots$

$r = -3$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{\cancel{16}^{\,4}\left(1-(-3)^{10}\right)}{1+(+3)}$$

$= 4(1 - (59049))$

$= 4(-59048)$

$= \boxed{-236192}$

$3^{10} = 3^5 \cdot 3^5$

$= 243 \cdot 243$

$$\begin{array}{r} 243 \\ \times 243 \\ \hline 729 \\ 9720 \\ 48600 \\ \hline 59049 \end{array}$$

You try...

$a_n = a_1 r^{n-1}$

$\sqrt[3]{27} = \sqrt[3]{1\, r^3}$

$\boxed{r = 3}$

① Write a sequence with two geometric means between 1 and 27.

$\boxed{1,\ 3,\ 9,\ 27}$

② Find the sum of the first 9 terms of the series

$+0.5 - 1 + 2 - \ldots$

$\boxed{r = -2}$

$\boxed{85.5 \text{ or } \frac{171}{2}}$

HW p 771, 772

(17 - 39) odd

$r = -2$

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad S_9 = \frac{\frac{1}{2}\left(1-(-2)^9\right)}{1+(+2)}$$

$$= \frac{\frac{1}{2}(1+ +512)}{3} = \frac{\frac{1}{2}(5\cancel{1}3)}{\cancel{3}} = \frac{1}{2}(171)$$

$$(-2)^9 = -512$$

$$\boxed{= \frac{171}{2}}$$

$$\begin{array}{r} 171 \\ 3\overline{)513} \\ -3 \\ \hline 21 \end{array}$$

$$= 85\tfrac{1}{2}$$

$$= 85.5$$