Sect. 10-4: The Hyperbola

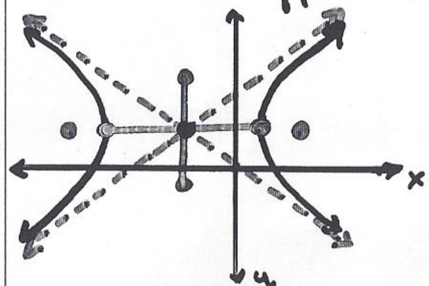
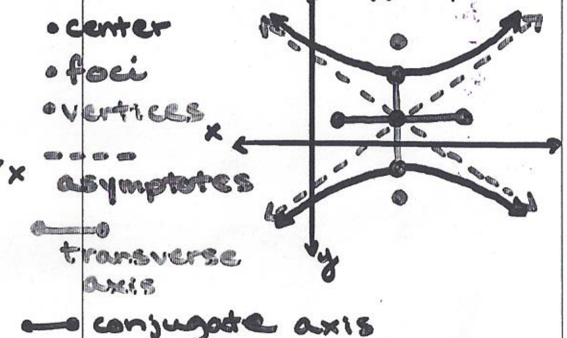
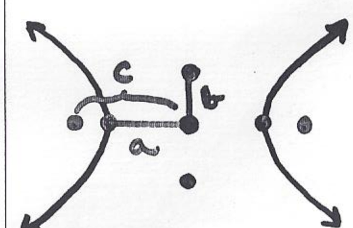
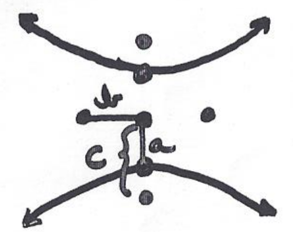
An **hyperbola** is the locus of all points in a plane such that the absolute value of the differences of the distance from two given points in the plane, called **foci**, is constant.

*The midpoint of the segment connecting the foci is called the **center**.
*The point on each branch of the hyperbola nearest the center is called a **vertex**.
*The **asymptotes** of a hyperbola are lines that the curve approaches as it recedes from the center.

A hyperbola has two **axes of symmetry**.
*TRANSVERSE AXIS - the line segment that has its endpoints at the vertices (like the major axis)
*CONJUGATE AXIS - the segment perpendicular to transverse axis through the center

a = the distance from the center to either vertex
[on the transverse (major) axis]
b = the distance from the center to the endpoints of the conjugate (minor) axis
c = the distance from the center to either focus

The relationship between a, b, and c is
$a^2 = c^2 - b^2$.

HORIZONTAL HYPERBOLA	VERTICAL HYPERBOLA
transverse axis ‖ to x-axis	transverse axis ‖ to y-axis
	
center (h,k)	center (h,k)
$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
equation of asymptotes: $y-k=\pm\frac{b}{a}(x-h)$	equation of asymptotes: $y-k=\pm\frac{a}{b}(x-h)$
	

NOTES:
*a is always first (so is the "major" axis)
*the numerator of the slope is always the square root of the value under the "y"

If a = b, then the hyperbola is an **equilateral hyperbola** and its asymptotes are perpendicular.

*if the equation is in the form $xy = c$ ($c \neq 0$), then the x- and y-axes are asymptotes

*if $c>0$, then the branches lie in Quadrants I and III

*if $c<0$, then the branches lie in Quadrants II and IV

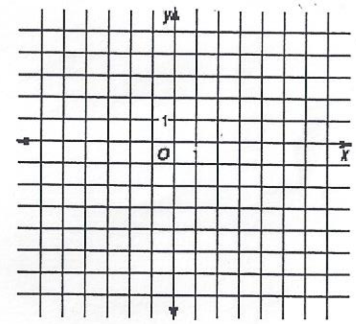
Compare and contrast the general equations of circles, ellipses, and hyperbolas.

Examples:

1) Write the equation of the hyperbola if the foci are located at (4,0) and (-4,0), and the vertices are located at (1,0) and (-1,0).

Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of each hyperbola. Then graph each hyperbola.

2) $\frac{(x-5)^2}{25}-\frac{(y+1)^2}{9}=1$



3) $4x^2 - y^2 + 24x + 4y + 28 = 0$