Infinite Sequences + Series

Note Title 9/14/2010

Objective: To 1) determine the limit of the terms of an infinite sequence, and 2) find the sum of an infinite geometric series.

Paper Coloring Activity

$$\boxed{\lim_{n\to\infty} \frac{1}{n} = 0}$$

Ex1 Limits at Infinity

$$\lim_{x\to\infty} \frac{1}{x+1} \qquad \frac{0}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{11}, \ldots \frac{1}{101} \ldots$$

$$\frac{1}{1{,}000{,}001} \longrightarrow 0 \qquad \boxed{= 0}$$

HA $\quad N < D$

$y = 0$

$\lim_{x \to \infty} \frac{x^2}{x^1+1}$ $\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \ldots \frac{10000}{101}$ DNE

Does not exist.

N > D

No HA

HA!

$\lim_{x \to \infty} \frac{\boxed{1}x^2}{\boxed{2}x^2-1} = \frac{1}{1}, \frac{4}{7}, \frac{9}{17}, \frac{16}{31}, \ldots \frac{100}{199}$

$\ldots, \frac{1000000}{1999999} \longrightarrow \frac{1}{2}$

$\boxed{= \frac{1}{2}}$

"Rules" for Evaluating Limits at Infinity

<u>N</u> = degree of numerator   <u>D</u> = degree of denominator

<u>degree</u>: highest exponential power

① $N < D$; $\lim\limits_{x \to \infty} f(x) = 0$

② $N > D$; $\lim\limits_{x \to \infty} f(x) =$ Does Not Exist (DNE)

③ $N = D$; $\lim\limits_{x \to \infty} f(x) =$ ratio of lead coefficients

<u>Note</u>: These follow the horizontal asymptote rules since HAs <u>LIMIT</u> the height of a function!

Ex2 Find each limit.

a) $\lim_{n\to\infty} \frac{\boxed{3}n^1+6}{\boxed{1}n^1} = 3$

b) $\lim_{n\to\infty} \frac{\boxed{1}n^2-3n+4}{\boxed{1}n^2-1} = \boxed{1}$

c) $\lim_{n\to\infty} \frac{4n^2-6}{3n^1} = \text{DNE}$

N > D

d) $\lim_{n\to\infty} \frac{2n^2-1}{n^3+1} = 0$

N < D

e) $\lim_{n\to\infty} \frac{(-1)^n n}{5n^1+1}$ $-$ $+$ $-$ $+$ $= \text{DNE}$

oscillating f(x)

* $S = \dfrac{a_1}{1-r}$ if $\boxed{-1 < r < 1}$ } Sum of an Infinite Geometric Series

Ex3: Find the sum.

a) $\underset{a_1}{60} + 24 + 9.6 + \ldots$

$r = \dfrac{24}{60} = \boxed{\dfrac{2}{5}}$

$S = \dfrac{60}{\frac{5}{5} - \frac{2}{5}} = \dfrac{60}{\frac{3}{5}} \left[\dfrac{5}{3}\right]$

$60 \cdot \dfrac{5}{3} = \dfrac{300}{3} = 100$

b) $1 + \boxed{\dfrac{5}{2}} + \dfrac{25}{4} + \ldots$

$1 \quad 2.5 + 6.25$

$r = \dfrac{5}{2}$

No sum!

Repeating decimal

infinite geometric sum

$5.\boxed{762762762}\ldots$

Ex4: a) Write $5.\overline{762}$ as a fraction.

$5\frac{254}{333}$

$$\frac{762}{1000} + \frac{762}{1{,}000{,}000} + \frac{762}{1{,}000{,}000{,}000} + \ldots$$

$$S = \frac{a_1}{1-r} = \frac{\frac{762}{1000}}{1 - \frac{1}{1000}}$$

$$r = \frac{1}{1000}$$

$$\boxed{\frac{254}{333}} = \frac{762}{999}$$

$$= \frac{\frac{762}{\cancel{1000}}}{\frac{999}{\cancel{1000}}} \left[\frac{\cancel{1000}}{999}\right] \left[\frac{1000}{999}\right]$$

$$3\overline{)762} = 254$$

6

16

15

12

b) Write $0.9\overline{222}$ as a fraction.

0.9

$$\frac{9}{10}\left(\frac{9}{9}\right)$$

$$\boxed{\frac{81}{90}}$$

$$\frac{2}{100} + \frac{2}{1000} + \frac{2}{10000} + \ldots \quad \boxed{\frac{83}{90}}$$

$$r = \frac{1}{10}$$

$$S = \frac{a_1}{1-r} = \frac{\frac{2}{100}}{1 - \frac{1}{10}} = \frac{\frac{2}{\cancel{100}\,10}}{\frac{9}{10}} \left[\frac{\cancel{10}}{9}\right] = \boxed{\frac{2}{90}}$$

$$\left[\frac{10}{9}\right]$$

Ex 5: A rubber ball is dropped to the floor from a height of [illegible]. If the ball rises at each rebound to a height of $\boxed{\frac{2}{3}}$ its previous height. Find the total distance the ball travels before coming to rest.

$$\underset{a_1}{27} + 18 + 12 + 8 \ldots + \frac{16}{3}$$

$$r = \frac{2}{3}$$

$$S = \frac{27}{1-\frac{2}{3}} = \frac{27\left[\frac{3}{1}\right]}{\frac{1}{3}\left[\frac{3}{1}\right]} = 81$$

$$S = \frac{a_1}{1-r}$$

HW p781 (14-22, 25-37)
all odd