

# Infinite Sequences + Series

Objective: To 1) determine the limit of the terms of an infinite sequence, and 2) find the sum of an infinite geometric series.

Paper Coloring Activity

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

## Ex1 Limits at Infinity

$\sum_{x \rightarrow \infty} \frac{1}{x+1}$

$0 \quad \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{11}, \dots, \frac{1}{101}, \dots$

$N < D$

$H \neq y = 0$

$\left| \frac{1}{1,000,001} \right| \rightarrow 0$

$\boxed{= 0}$

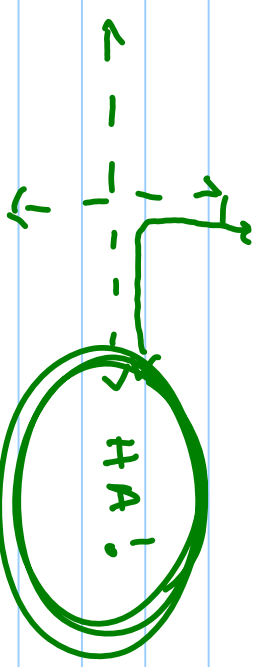
lim  
 $x \rightarrow \infty$

$$\frac{x^2}{x+1}$$

- $\frac{1}{2}$
- $\frac{4}{3}$
- $\frac{9}{4}$
- ...

$$\frac{10000}{101}$$

$N > D$   
No HA



DNE  
Does not exist.

$$\lim_{x \rightarrow \infty} \frac{x}{2x-1}$$

- $\frac{1}{1}$
- $\frac{4}{7}$
- $\frac{9}{17}$
- $\frac{16}{31}$
- ...
- $\frac{100}{199}$

$$\frac{1000000}{1999999}$$

$\frac{1}{2}$

$$= \frac{1}{2}$$

## "Rules" for Evaluating Limits at Infinity

N = degree of numerator     D = degree of denominator  
degree: highest exponential power

$$\textcircled{1} N < D ; \lim_{x \rightarrow \infty} f(x) = 0$$

$$\textcircled{2} N > D ; \lim_{x \rightarrow \infty} f(x) = \text{Does Not Exist (DNE)}$$

$$\textcircled{3} N = D ; \lim_{x \rightarrow \infty} f(x) = \text{ratio of lead coefficients}$$

Note: These follow the horizontal asymptote rules since  
HAs LIMIT the height of a function!

Ex 2 Find each limit.

a)  $\lim_{n \rightarrow \infty}$

$$\boxed{3} \frac{n^1 + 6}{\boxed{1} n^1} = 3$$

b)  $\lim_{n \rightarrow \infty}$

$$\boxed{1} \frac{n^2 - 3n + 4}{\boxed{1} n^2 - 1} = \boxed{1}$$

c)  $\lim_{n \rightarrow \infty}$

$$4n^2 - 6 = \text{DNE}$$

d)  $\lim_{n \rightarrow \infty}$

$$\frac{2n^2 - 1}{n^3 + 1} = 0$$

$N > D$

$N < D$

e)  $\lim_{n \rightarrow \infty} \frac{(-1)^n n}{5n^2 + 1}$

$\ominus$

$\oplus$

$\ominus$

$\oplus$

$= \text{DNE}$

Oscillating  $f(x)$

$$S = \frac{a_1}{1-r}$$

if

$$-1 < r < 1$$

Sum of an  
Infinite  
Geometric Series

Ex3: Find the sum.

a)  $100 + 24 + 9.6 + \dots$

$$r = \frac{24}{100} = \frac{2}{5}$$

$$S = \frac{100}{1 - \frac{2}{5}} = 100 \left[ \frac{5}{3} \right]$$

$$100 \cdot \frac{5}{3} = \frac{300}{3} = 100$$

b)  $1 + \frac{5}{2} + \frac{25}{4} + \dots$

$$r = \frac{5}{2}$$

No sum

Repeating decimal  
infinite geometric sum

5.  $\overline{762762762} \dots$

Ex: Write  $5.\overline{762}$  as a fraction.

$$\frac{762}{1000} + \frac{762}{1,000,000} + \frac{762}{1,000,000,000} + \dots$$

$$S = \frac{a_1}{1-r} = \frac{762}{1-0.001}$$

$$r = \frac{1}{1000} \quad \frac{254}{333} = \frac{762}{999} = \frac{762}{1000} \left[ \frac{1000}{999} \right] = \frac{762}{1000} \left[ \frac{1000}{999} \right]$$

Write  $0.\overline{9222}$  as a fraction.

$$0.9 \frac{2}{10} \left( \frac{9}{9} \right) \quad \frac{2}{100} + \frac{2}{1000} + \frac{2}{10000} + \dots$$

$$\frac{83}{90} \quad \frac{2}{10} \left[ \frac{10}{9} \right] = \frac{2}{90}$$

Ex 5: A rubber ball is dropped to the floor from a height of  $27$  m. If the ball rises at each rebound to a height of  $\frac{2}{3}$  its previous height. Find the total distance the ball travels before coming to rest.

$$27 + 18 + 12 + 8 \dots + \frac{16}{3}$$
$$a_1$$

$$r = \frac{2}{3}$$

$$S = \frac{27}{1 - \frac{2}{3}} = \frac{27}{\frac{1}{3}} \left[ \frac{3}{1} \right] = 81$$

$$S = \frac{a_1}{1-r}$$

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HW P 781 (14-22, 25-37)  
    <sub>even</sub>          <sub>odd</sub>