Name ___________________________________

CALCULUS: LIMITS, CONTINUITY & DIFFERENTIABILITY PACKET

1. Explain in your own words what is meant by the equation

$$\lim_{x \to 2} f(x) = 5.$$

Is it possible for this statement to be true and yet $f(2) = 3$? Explain.

2. Explain what it means to say that

$$\lim_{x \to 1^-} f(x) = 3 \text{ and } \lim_{x \to 1^+} f(x) = 7.$$

In this situation, it is possible that $\lim_{x \to 1} f(x)$ exists?

3. Explain the meaning of each of the following.

(a) $\lim_{x \to -3} f(x) = \infty$

(b) $\lim_{x \to 4^+} f(x) = -\infty$

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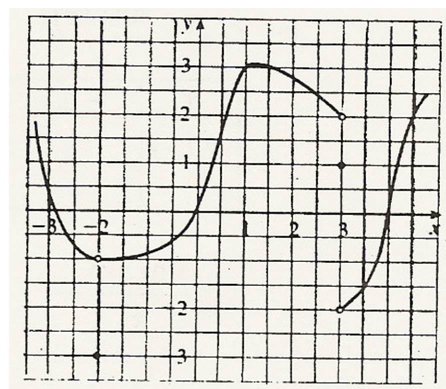
4. For the function f whose graph is given, state the value of the given quantity, if it exists. If it does not exist, explain why.



(a) $\lim_{x \to 1} f(x) =$ (b) $\lim_{x \to 3^-} f(x) =$ (c) $\lim_{x \to 3^+} f(x) =$

(d) $\lim_{x \to 3} f(x) =$ (e) $f(3) =$ (f) $\lim_{x \to -2^-} f(x) =$

(g) $\lim_{x \to -2^+} f(x) =$ (h) $\lim_{x \to -2} f(x) =$ (i) $f(-2) =$

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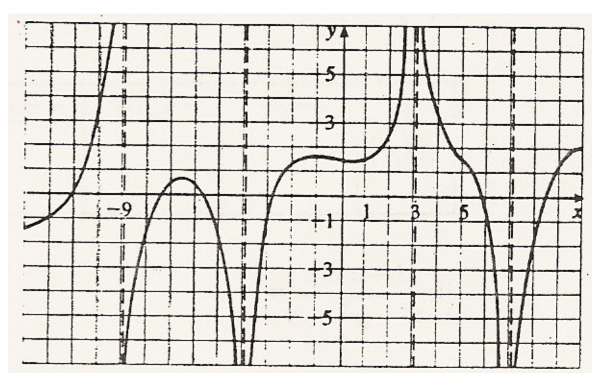
5. For the function f whose graph is shown, state the following.



(a) $\lim_{x \to 3} f(x) =$ (b) $\lim_{x \to 7} f(x) =$

(c) $\lim_{x \to -4} f(x) =$ (d) $\lim_{x \to -9^-} f(x) =$

(e) $\lim_{x \to -9^+} f(x) =$

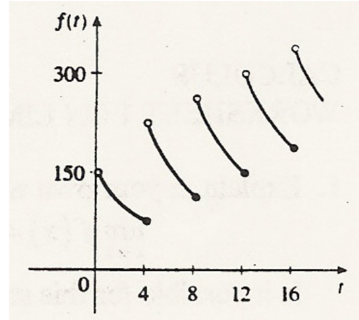
(f) The equations of the vertical asymptotes

TURN--->>>

6. A patient receives a 150-mg injection of a drug every four hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find

$$\lim_{t\to 12^-} f(t) \text{ and } \lim_{x\to 12^+} f(t)$$

and explain the significance of these one-sided limits.



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7. Sketch the graph of the function $f(x)=\dfrac{1}{\left(1+2^{1/x}\right)}$ and state the value of each limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 0^-} f(x)=$ (b) $\lim_{x\to 0^+} f(x)=$ (c) $\lim_{x\to 0} f(x)=$

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8. Sketch the graph of the following function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x)=\begin{cases} 2-x, & x<-1 \\ x, & -1\le x<1 \\ (x-1)^2, & x\ge 1 \end{cases}$$

Fill in the table for the following functions to find the given limit.

9. $f(x)=\dfrac{\sin(3x)}{x}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$							

$$\lim_{x\to 0}\frac{\sin(3x)}{x}=$$

10. $g(x)=\dfrac{1-\cos x}{x^2}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$g(x)$							

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=$$

TURN--->>>

11. Given that $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$, $\lim_{x\to a} h(x) = 8$, find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to a}\left[f(x)+h(x)\right]=$

(b) $\lim_{x\to a}\left[f(x)\right]^2 =$

(c) $\lim_{x\to a}\sqrt[3]{h(x)} =$

(d) $\lim_{x\to a}\frac{1}{f(x)} =$

(e) $\lim_{x\to a}\frac{f(x)}{h(x)} =$

(f) $\lim_{x\to a}\frac{g(x)}{f(x)} =$

(g) $\lim_{x\to a}\frac{f(x)}{g(x)} =$

(h) $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)} =$

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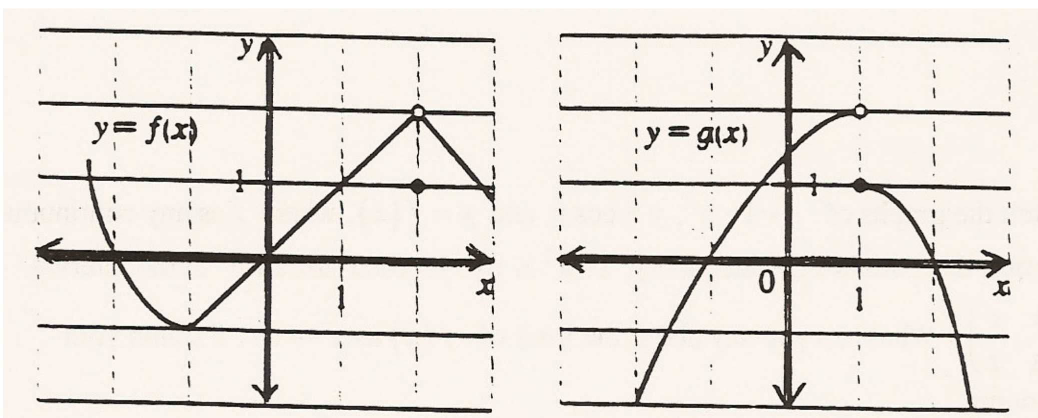
12. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.



(a) $\lim_{x\to 2}\left[f(x)+g(x)\right]=$

(b) $\lim_{x\to 1}\left[f(x)+g(x)\right]=$

(c) $\lim_{x\to 0}\left[f(x)g(x)\right]=$

(d) $\lim_{x\to -1}\frac{f(x)}{g(x)} =$

(e) $\lim_{x\to 2} x^3 f(x) =$

(f) $\lim_{x\to 1}\sqrt{3+f(x)} =$

Find the limit. Draw a sketch for each problem. Do not use your calculator.

13. $\lim_{x\to 1^+}\frac{1}{x-1} =$

14. $\lim_{x\to 1}\frac{1}{x-1} =$

15. $\lim_{x\to -3}\frac{1}{(x+3)^2} =$

TURN--->>>

16. $\lim_{x\to 5^-} \frac{1}{5-x} =$

17. $\lim_{x\to 5^-} \frac{1}{(5-x)^2} =$

18. $\lim_{x\to 2} \frac{-1}{(x-2)^2} =$

19. $\lim_{x\to 3} \frac{|x-3|}{x-3} =$

20. $\lim_{x\to 2} [x+1] =$

21. $\lim_{x\to 2^+} \frac{x^3|x-2|}{x-2} =$

22. $\lim_{x\to 4^-} \frac{x^3[x-4]}{x-4} =$

23. $\lim_{x\to 3^+} \left(x-3-\frac{1}{x-3}\right) =$

24. $\lim_{x\to \frac{\pi}{2}^+} \tan x =$

25. $\lim_{x\to -\frac{\pi}{2}^+} \sec x =$

26. $\lim_{x\to \pi^-} \csc x =$

27. $\lim_{x\to 0^-} \cot x =$

28. $f(x) = \begin{cases} x^2 - 1 \text{ if } x < 2 \\ 3x - 2 \text{ if } x > 2 \end{cases}$

$a)\ \lim_{x\to 2^-} f(x) =$

$b)\ \lim_{x\to 2^+} f(x) =$

$c)\ \lim_{x\to 2} f(x) =$

29. $g(x) = \begin{cases} x - 3 \text{ if } x \neq 1 \\ 4 \text{ if } x = 1 \end{cases}$ $\lim_{x\to 1} g(x) =$

30. $h(x) = \begin{cases} x + 3 \text{ if } x < 1 \\ 3x^2 + 1 \text{ if } x > 1 \end{cases}$ $\lim_{x\to 1} h(x) =$

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31. Determine if the following statements regarding the function $y = f(x)$ are true or false.

a. $\lim_{x\to -1^+} f(x) = 1$

b. $\lim_{x\to 0^-} f(x) = 0$

c. $\lim_{x\to 0^-} f(x) = 1$

d. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

e. $\lim_{x\to 0} f(x)$ exists

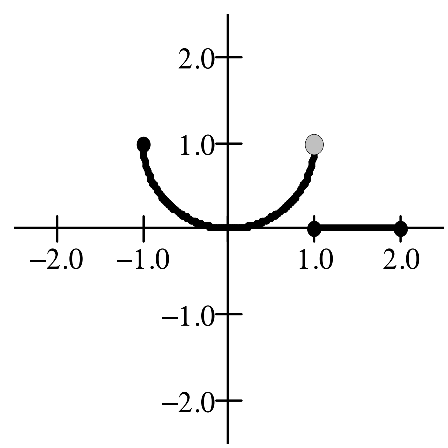
f. $\lim_{x\to 0} f(x) = 0$

g. $\lim_{x\to 0} f(x) = 1$

h. $\lim_{x\to 1} f(x) = 1$

i. $\lim_{x\to 1} f(x) = 0$

j. $\lim_{x\to 2} f(x) =$



TURN--->>>

On problems 32 - 37 (Use your graphing calculator on problems 36 and 37):

(a) find $\lim_{x\to\infty} f(x)$

(b) find $\lim_{x\to-\infty} f(x)$

(c) identify all horizontal asymptotes.

32. $f(x)=\dfrac{3x^3-x+1}{x+3}$

33. $f(x)=\dfrac{4x^2-3x+5}{2x^3+x-1}$

34. $f(x)=\dfrac{3x+1}{x-4}$

35. $f(x)=\dfrac{3x+1}{|x|+2}$

36. $f(x)=\dfrac{\sin 3x}{x}$

37. $f(x)=\cos\left(\dfrac{1}{x}\right)$

On problems 38 - 41 (Use your calculator on problems 37 – 39):

(a) find the vertical asymptotes of $f(x)$

(b) describe the behavior of $f(x)$ to the left and right of each vertical asymptote.

38. $f(x)=\dfrac{1}{x^2-4}$

39. $f(x)=\dfrac{x^2+5x+6}{x^2-4}$

40. $f(x)=\dfrac{x^2-2x}{x+1}$

41. $f(x)=\sec x$

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42. Given $f(x)=\dfrac{2-5|x|}{3x+4}$, find the following. Show all work!

(a) $\lim_{x\to\infty} f(x)$

(b) $\lim_{x\to-\infty} f(x)$

(c) Name the horizontal asymptotes of *f*.

(d) Name the vertical asymptotes of *f*.

On problems 43 and 44:

(a) Find the values of *x*, if any, for which the function is discontinuous.

(b) Identify each discontinuity as point, jump, or asymptotic.

(c) Identify each discontinuity as removable or nonremovable.

43. $f(x)=\dfrac{x^2-9}{x-3}$

44. $g(x)=\dfrac{x-1}{x^2-x-2}$

TURN--->>>

For problems 45 - 47, graph the function. Determine if the function is continuous at the specified value of x. Justify (making sure to show all three steps of the definition).

45. $g(x)=\begin{cases} x+5 & x\neq 0 \\ 4 & x=0 \end{cases}$ at x = 0

46. $h(x)=\begin{cases} -x^2+8 & x<2 \\ \frac{1}{2}x+3 & x\geq 2 \end{cases}$ at x = 2

47. $f(x)=\begin{cases} \frac{x^2-1}{x-1} & x\neq 1 \\ x & x=1 \end{cases}$

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48. $f(x)=\begin{cases} \frac{x^2+3x-10}{x-2} & x\neq 2 \\ ? & x=2 \end{cases}$ Define $f(2)$ in a way that intends $f(x)$ to be continuous.

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49. Find *k* so that *f* will be continuous at *x* = 3, given $f(x)=\begin{cases} kx^2, & x\leq 2 \\ 2x+k, & x>2 \end{cases}$.

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50. Determine if $f(x)$ is continuous at $x=0$ and $x=1$. Justify. $f(x)=\begin{cases} x+1, & x<0 \\ e^x, & 0\leq x\leq 1 \\ 2-x, & x>1 \end{cases}$

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51. Find a value of constant k that will make the function continuous. $f(x)=\begin{cases} kx^2, & x\leq 2 \\ 2x+k, & x>2 \end{cases}$

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52. Find a value of constant k and m that will make the function continuous.

$$f(x)=\begin{cases} x^2+5, & x>2 \\ m(x+1)+k, & -1<x\leq 2 \\ 2x^3+x+7, & x\leq -1 \end{cases}$$

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53. Find a value of constant k that will make the function continuous. $f(x)=\begin{cases} \frac{\sin 3x}{x}, & x\neq 0 \\ k, & x=0 \end{cases}$

TURN--->>>

54. Find a value of constant c and d that will make the function continuous.

$$m(x)=\begin{cases} -\sqrt{4-(x+3)^2}, & x\le -1 \\ cx+d, & -1<x<3 \\ \sqrt{x-3}+4, & x\ge 3 \end{cases}$$

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55. Show whether the Intermediate Value Theorem holds. If the theorem holds, find the value of c which the theorem guarantees; if the theorem does not hold give the reason. Also sketch the graph of *f*.

$$f(x)=(x-3)^2+2,\ [a,b]=[1,4],\ k=5$$

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56. (Calculator allowed.) The population y, of bacteria *Makeyoucoughus hurtyourthruatus* is modeled by the equation $y=50e^{.1013663t}$, where t is days and y is the number of colonies of bacteria. Use the Intermediate Value Theorem to verify that the bacteria will reach a population of 100 colonies on the time interval [4,7]. Then determine when the population will reach 100 colonies.

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57. Just for fun….

James' Diabolical Challenge Problem:

Given $j(x)=\begin{cases} \dfrac{x^2-(4+A)x+4A}{x-4}, & x\ne 4 \\ B, & x=4 \end{cases}$ and $j(2)=1$ and $j(x)$ is everywhere continuous,

find A and B.

☺ You've Finished Your First Packet of Calculus! ☺