The following problems are maximum/minimum optimization problems. They illustrate one of the most important applications of the first derivative. Many students find these problems intimidating because they are "word" problems, and because there does not appear to be a pattern to these problems. However, if you are patient you can minimize your anxiety and maximize your success with these problems by following these guidelines:

GUIDELINES FOR SOLVING MAX/MIN PROBLEMS

1. Read each problem slowly and carefully. Read the problem at least three times before trying to solve it. Sometimes words can be ambiguous. It is imperative to know exactly what the problem is asking. If you misread the problem or hurry through it, you have NO chance of solving it correctly.

2. If appropriate, draw a sketch or diagram of the problem to be solved. Pictures are a great help in organizing and sorting out your thoughts.

3. Define variables to be used and carefully label your picture or diagram with these variables. This step is very important because it leads directly or indirectly to the creation of mathematical equations.

4. Write down all equations which are related to your problem or diagram. Clearly denote that equation which you are asked to maximize or minimize. Experience will show you that MOST optimization problems will begin with two equations. One equation is a "constraint" equation and the other is the "optimization" equation. The "constraint" equation is used to solve for one of the variables. This is then substituted into the "optimization" equation before differentiation occurs. Some problems may have NO constraint equation. Some problems may have two or more constraint equations.

5. Before differentiating, make sure that the optimization equation is a function of only one variable. Then differentiate using the well-known rules of differentiation.

6. Verify that your result is a maximum or minimum value using the first or second derivative test for extrema.

PROBLEM 1: Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

PROBLEM 2: Build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen?

PROBLEM 3: An open rectangular box with square base is to be made from 48 $ft.^2$ of material. What dimensions will result in a box with the largest possible volume?

PROBLEM 4: A container in the shape of a right circular cylinder with no top has surface area 3π $ft.^2$ What height h and base radius r will maximize the volume of the cylinder?

PROBLEM 5: A sheet of cardboard 3 ft. by 4 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume?

PROBLEM 6: Find the point (x, y) on the graph of $y = \sqrt{x}$ nearest the point (4, 0).