Sect. 12-4: Convergent and Divergent Series

If an infinite series has a sum, or limit, the series is <u>CONVERGENT</u>. If a series is not convergent, it is <u>DIVERGENT</u>.

Are arithmetic series convergent or divergent?

Are geometric series convergent or divergent?

<u>**Examples**</u>: Determine whether each series is convergent or divergent.

1) $-6 - 3 - 0 + 3 + \ldots$

2) $2 + 10 + 50 + \ldots$

3) $\frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \ldots$

4) $-8 - 2 + 4 + \ldots$

5) $1 + 5 + 25 + \ldots$

If a series is neither arithmetic nor geometric, we must have a different way to decide its convergence or divergence.

Ratio Test for Convergence of a Series

Let a_n and a_{n+1} represent two consecutive terms of a series of positive terms. Suppose $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists and that $r = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Then the series is convergent if $r < 1$ & divergent if $r > 1$. If $r = 1$, the test provides no information.

Examples: Use the ratio test to determine whether each series is convergent or divergent.

1) $2 + \frac{2^2}{2} + \frac{2^3}{3} + \ldots$

2) $1+\frac{1}{2^2}+\frac{1}{3^3}+\ldots$

3) $2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\ldots$

Remember: "n!" is read "n factorial" and means

$$n! = (n)(n-1)(n-2)\ldots(1)$$

4) $1+\frac{2}{1\cdot 2\cdot 3}+\frac{3}{1\cdot 2\cdot 3\cdot 4\cdot 5}+\frac{4}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7}+\ldots$

You try:

1) $10+\frac{10^2}{1\cdot 2}+\frac{10^3}{1\cdot 2\cdot 3}+\ldots$

2) $1+\frac{1}{2!}+\frac{1}{3!}+\ldots$