Review Index

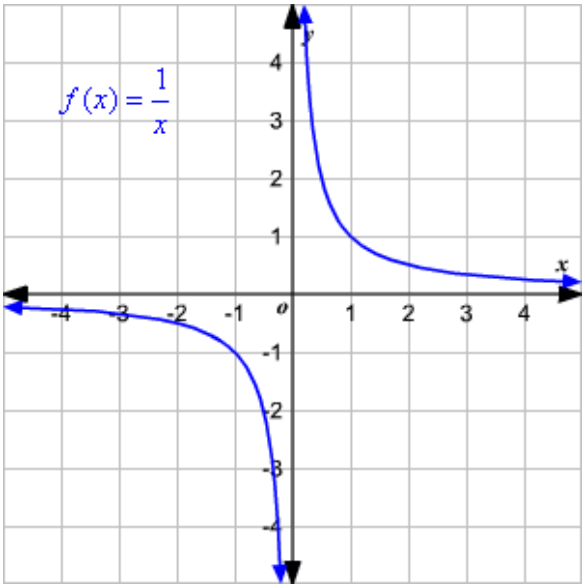
Rational Functions

A rational function is defined as the quotient of polynomials in which the denominator has a degree of at least 1. In other words, there must be a variable in the denominator.

The general form of a rational function is $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

Examples: $y = \frac{3}{x}, y = \frac{2x+1}{x+5}, y = \frac{1}{x^2}$

The parent function of a rational function is $f(x) = \frac{1}{x}$ and the graph is a hyperbola.



The domain and range is the set of all real numbers except 0.

$\text{Domain} : \{x | x \neq 0\}$

$\text{Range} : \{y | y \neq 0\}$

Excluded value

In a rational function, an excluded value is any x-value that makes the function value y undefined. So, these values should be excluded from the domain of the function.

For example, the excluded value of the function $y = \frac{2}{x+3}$ is –3. That is, when $x = -3$, the value of y is undefined.

So, the domain of this function is set of all real numbers except –3.

Asymptotes

An asymptote is a line that the graph of the function approaches, but never touches. In the parent function

$f(x) = \frac{1}{x}$, both the x - and y -axes are asymptotes. The graph of the parent function will get closer and closer to but never touches the asymptotes.

A rational function in the form $y = \frac{a}{x-b} + c$ has a vertical asymptote at the excluded value, or $x = b$, and a horizontal asymptote at $y = c$.