

Related Rates

Note Title

12/14/2012

Ex 1a) $\frac{dh}{dt} = 3 \frac{\text{in}}{\text{yr}}$

1b) $\frac{dm}{dt} = -4 \text{ cents/day}$

1c) $\frac{dr}{dt} = 4 \text{ ft/hr}$

1d) $\frac{dV}{dt} = -2 \text{ in}^3/\text{sec}$

2)
$$\begin{array}{|l} P = 32 \\ A = 60 \end{array}$$
 6
10

$$\begin{array}{|l} P = 40 \\ A = 96 \end{array}$$
 8
12

$$\begin{array}{|l} P = 48 \\ A = 140 \end{array}$$
 10
14

$$P = 2l + 2w$$

$$A = lw$$

$$\frac{dP}{dt} = 2 \frac{dl}{dt} + 2 \frac{dw}{dt}$$

$$\frac{dA}{dt} = l \frac{dw}{dt} + w \frac{dl}{dt}$$

$$a) \frac{dl}{dt} = \underline{\underline{2 \text{ in/sec}}}$$

$$\frac{dw}{dt} = \underline{\underline{2 \text{ in/sec}}}$$

$$\begin{aligned} \frac{dP}{dt} &= 2(2) + 2(2) \\ &= 4 + 4 \\ &= 8 \text{ in/sec} \end{aligned}$$

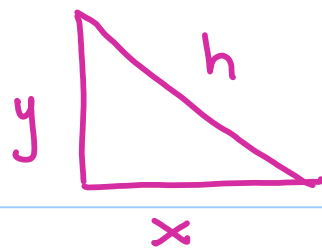
$$b) \frac{de}{dt} = -2 \text{ in/sec}$$

$$\frac{dw}{dt} = -2 \text{ in/sec}$$

$$\begin{aligned} \frac{dA}{dt} &= 10(2) + 6(2) \\ &= 20 + 12 \\ &= 32 \text{ in}^2/\text{sec} \end{aligned}$$

$$\begin{aligned} \frac{dA}{dt} &= 10(3) + 6(-3) \\ &= 30 - 18 \\ &= 12 \text{ in}^2/\text{sec} \end{aligned}$$

$$A = \frac{1}{2}xy$$



$$\frac{dh}{dt} = ?$$

$$x^2 + y^2 = h^2$$

$$h = \sqrt{x^2 + y^2}$$

$$\frac{dA}{dt} = \frac{1}{2} \left[x \frac{dy}{dt} + y \frac{dx}{dt} \right]$$

$$\frac{dh}{dt} =$$

x = short side

y = long side

$$\frac{dx}{dt} = 3 \text{ in/sec}$$

$$\frac{dy}{dt} = \underline{-5 \text{ in/sec}}$$

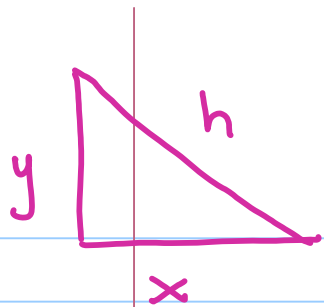
$$\frac{dA}{dt} = \frac{-30}{2}$$

$$= -15 \text{ in}^2/\text{sec}$$

$$\frac{dA}{dt} = \frac{1}{2} [30(-5) + 40(3)]$$

$$= \frac{1}{2} (-150 + 120) = \frac{1}{2} (-30) =$$

$$\boxed{135 \text{ in}^2/\text{sec}}$$



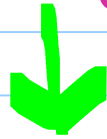
$$\frac{dh}{dt} = ?$$

$$h = (x^2 + y^2)^{\frac{1}{2}}$$

$$x^2 + y^2 = h^2$$

$$h = \sqrt{x^2 + y^2}$$

$$\frac{dh}{dt} = \frac{1}{2} (x^2 + y^2)^{-\frac{1}{2}} \left[x \frac{dx}{dt} + y \frac{dy}{dt} \right]$$



$$\frac{dh}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$$

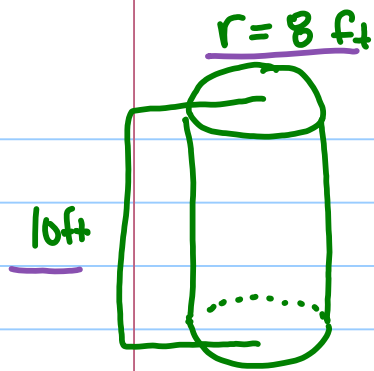
$$\frac{dx}{dt} = 3 \text{ in/sec}$$

$$\frac{dy}{dt} = 5 \text{ in/sec}$$

$$\frac{dh}{dt} = \frac{30(3) + 40(5)}{\sqrt{30^2 + 40^2}}$$

$$= \frac{90 + 200}{\sqrt{2500}}$$

$$= \frac{290}{50} = \frac{29}{5} \text{ in/sec}$$



a)

$\frac{164}{3}$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi \left[r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt} \right]$$

$$= \pi \left(8^2 (-3) + 10 (2 \cdot 8) 2 \right)$$

$$= \pi (-192 + 320)$$

$$= 128\pi \text{ ft}^3/\text{min}$$

$$\frac{dr}{dt} = 2$$

$$\frac{dh}{dt} = -3$$

$$SA = 2\pi r^2 + 2\pi r h$$

$$\frac{dSA}{dt} = 2\pi \left[2r \frac{dr}{dt} + r \frac{dh}{dt} + h \frac{dr}{dt} \right]$$

$$= 2\pi \left[2(8)(2) + 8(-3) + 10(2) \right] = 2\pi (28) = 56\pi \frac{\text{ft}^2}{\text{min}}$$

$$\frac{dr}{dt} = 50 \text{ ft/min}$$

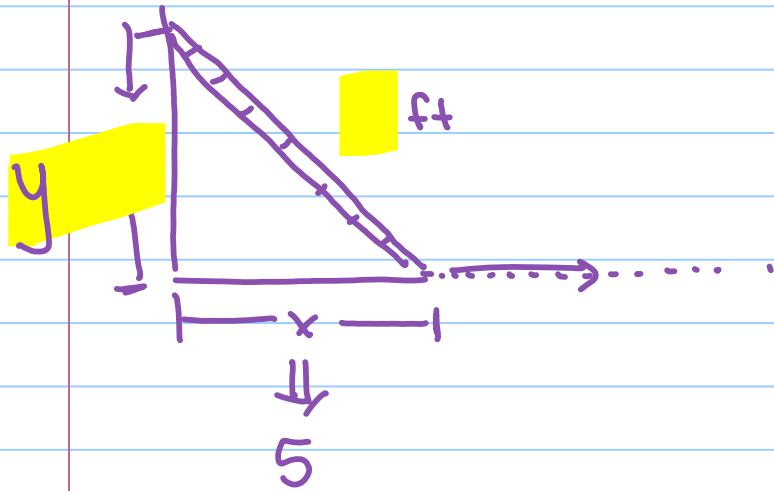
$$C = 2\pi r$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi(50) = 100\pi \text{ ft/min}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(20)(50) = 2000\pi \text{ ft}^2/\text{min}$$

$$= 2\pi(50)(50) = 5000\pi \text{ ft}^2/\text{min}$$



$$\frac{dx}{dt} = 2 \text{ ft/sec} \quad \frac{dy}{dt} = -$$

$$x^2 + y^2 = h^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2h \frac{dh}{dt}$$

$$\frac{dy}{dt} = -\frac{5}{6} \text{ ft/sec}$$

$$2(5)(2) + 2(12) \frac{dy}{dt} = 0 \quad = -\frac{5}{6}$$

$$20 + 24 \frac{dy}{dt} = 0 \quad \frac{dy}{dt} = -\frac{20}{24}$$

$$V = \frac{1}{3} \pi r^2 h$$