# RELATED RATES

<u>Objectives</u>: To understand the meaning of related rates and be able to use implicit differentiation to solve problems involving related rates.

- an important use of Chain Rule
- allow us to find the rates of change of two or more related variables that are changing with respect to time

<u>Example:</u> When a liquid is drained out of a conical shaped funnel the volume (V), radius (r) , and height (h) of the liquid level are all functions of time related by the equation

$$V = \frac{\pi}{3} r^2 h$$

You can differentiate implicitly to obtain the RELATED-RATE Equation.

$$\frac{dV}{dt} = \frac{\pi}{3}\left[ r^2 \frac{dh}{dt} = 2rh \frac{dr}{dt} \right]$$

The rate of change of the volume of liquid is related to the rates of change of BOTH the height and radius of the liquid level.

<u>EX 1:</u> Suppose x and y are both differentiable functions of t and are related by the equation

$$Y = x^2 + 3$$

Find $\frac{dy}{dt}$ when x = 1, given that $\frac{dx}{dt}$ = 2 when x = 1.

Equation:

Given rate:

Find:

EX 2:

Equation: $y = \sqrt{x}$

Given: $\frac{dx}{dt} = 3$

Find: $\frac{dy}{dt}$ when $x = 4$

Given: $\frac{dy}{dt} = 2$

Find: $\frac{dx}{dt}$ when $x = 25$

YOU TRY:

1a. Equation: $y = x^2 - 3x$

Given: $\frac{dx}{dt} = 1$

Find: $\frac{dy}{dt}$ when $x = 3$

2. Equation: $xy = 4$

Given: $\frac{dx}{dt} = 10$

Find: $\frac{dy}{dt}$ when $x = 8$

1b. Equation: $y = x^2 - 3x$

Given: $\frac{dy}{dt} = 5$

Find: $\frac{dx}{dt}$ when $x = 21$

2b. Equation: $xy = 4$

Given: $\frac{dy}{dt} = -6$

Find: $\frac{dx}{dt}$ when $x = 1$

HW:

Equation: $x^2 + y^2 = 25$

Given: $\frac{dx}{dt} = 8$

Find: $\frac{dy}{dt}$ when $x = 3, y = 4$

Equation: $x^2 + y^2 = 25$

Given: $\frac{dy}{dt} = -2$

Find: $\frac{dx}{dt}$ when $x = 4, y = 3$

GUIDELINES FOR SOLVING RELATED-RATE PROBLEMS

1. Identify all GIVEN quantities and quantities TO BE DETERMINED. Make a sketch and label the quantities.

2. Write an equation involving the variables whose rates of change either are given or are to be determined.

3. Using the Chain Rule, implicitly differentiate both sides of the equation WITH RESPECT TO TIME.

4. AFTER completing Step 3, substitute into the resulting equation all known values for the variable and their rates of change. Then solve for the required rate of change.

EX3: A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius, r, of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area, A, of the disturbed water changed?

Equation:

Given rate:

Find:

EX4: Air is being pumped into a spherical balloon at a rate of 4.5 cubic inches per minute. Find the rate of change of the radius when the radius is 2 inches.

Equation:

Given rate:

Find:

YOU TRY:

3. The radius of a circle is increasing at a rate of 2 centimeters per minute. Find the rate of change of the area when a) $r = 6$ cm and b) $r = 24$ cm.

4. A spherical balloon is inflated with gas at the rate of 500 cubic centimeters per minute. How fast is the radius changing at the instant the radius is a) 30 cm and b) 60 cm?