Series and Sequences : Arithmetic

Note Title 9/10/2010

Objective: To 1) find terms of an arithmetic sequence & 2) find the sum of an arithmetic series.

Sequence: a "collection" of numbers with something in common

Series: the SUM of a sequence

Arithmetic:

arithmetic pattern: adding a constant value repeatedly.

1, 2, 3, 4, 5, ... 6, 7, 8 (+1)

5, 10, 15, 20, 25, 30, 35, (+5)

(+6) a, $a+b$, $a+2b$, ... $a+3b$, $a+4b$, $a+5b$

17, 14, 11, 8, 5, 2, -1, -4, (-3)

$2\sqrt{2}, 1\sqrt{2}, 0, -1\sqrt{2}, -2\sqrt{2}, -3\sqrt{2}, \ldots$

$-2, -4, -6, -8, \ldots$ (-3)

$-\sqrt{3}$ $-4\sqrt{3}, -5\sqrt{2}$ $-10, -12, -14$

Ex 1: Find the 41st term in the arithmetic sequence $-12, -1, 10, \ldots$

$+11 \quad +11$ common difference

$$\text{term } 41 = -12 + 40(11)$$

$$= -12 + 440$$

$$= 428$$

Ex 1: Find the 41st term in the arithmetic sequence $-12, -1, 10, \ldots$

$$* \quad a_n = a_1 + (n-1)d$$

a_n : n^{th} or last term

a_1 = first term

n = number of terms

d = common difference

a = term

$a_{41} = -12 + (41-1)11$

$= -12 + 40(11)$

$= 428$

Ex2 : Find the [redacted] in the arithmetic sequence for which $a_{44} = 229$ and $d = 8$.

$a_1 = 229 + 43(-8)$

$a_1 = 229 - 344$

$\boxed{a_1 = -115}$

$a_n = a_1 + (n-1)d$

$229 = a_1 + (44-1)(8)$

$229 = a_1 + (+344)$

$-344 \qquad\qquad -344$

$\boxed{-115 = a_1}$

EX3 Write an arithmetic sequence that has 7 arithmetic means -12 and 23.

$-12, \wedge\ -7, \wedge\ -2, \wedge\ 3, \wedge\ 8, \wedge\ 13, \wedge\ 18, \wedge\ 23$

-12 ——— 35 ——— 23

$\frac{35}{7} = 5$

$a_n = a_1 + (n-1)d$

$23 = -12 + (8-1)d$

$+12 \qquad +12$

$\frac{35}{7} = \frac{7d}{7}$

$d = 5$

Karl Freidrich Gauss

EX4: Find the sum of all the numbers 1 to 100.

$1 + 2 + 3 + \ldots + 96 + 97 + 98 + 99 + 100$

$50(100) = 5000$

$5000 + 50 = 5050$

101
101

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$= 50(101)$

$= 5050$

$$S_{100} = \frac{100}{2}(1 + 100)$$

$= 50(101) = 5050$

Never ASSUME!

Ex 5: Find the sum of the first 32 terms in the arithmetic series $-12 \quad -6 \quad -0 \quad \ldots \quad 174$

(+6, +6)

$a_{32} = -12 + (31)6$

$= -12 + 186$

$= 174$

$S_{32} = 16(174 + 12)$

$= 16(162)$

$S = 2592$

$$\begin{array}{r} 3162 \\ \times \quad 16 \\ \hline 9726 \\ 1620 \\ \hline 25920 \end{array}$$

$$S_{32} = \frac{32}{2}(-12 + 174)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

"Sum"

Ex 6: Evaluate: $\sum_{k=3}^{10} (2k+1)$

Expand

$= 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$

$\sum$ sigma = sum

$= \frac{8}{2}(7 + 21)$

$= 4(28)$

$= 112$

HW p 763 (17-47) odd