

Series and Sequences : Arithmetic

Objective: To 1) find terms of an arithmetic sequence & 2) find the sum of an arithmetic series.

Sequence: a "collection" of numbers with something in common

Series: the SUM of a sequence

Arithmetic :

arithmetic pattern : adding a constant value repeatedly.

$1, 2, 3, 4, 5, \dots, 6, 7, 8$	$a + 2b, a + 4b, \dots, a + 3b, a + 5b$	$5, 10, 15, 20, 25, 30, 35, \dots$
$a, a + b, a + 2b, \dots, a + 3b, a + 5b$	$2\sqrt{a}, \sqrt{2}, 0, -\sqrt{2}, -2\sqrt{a}, -3\sqrt{2}, \dots$	$8, 5, 2, -1, -4, \dots$

(+1)
(+5)
(-3)
(-3)

$$\boxed{-\sqrt{3}}$$

$$-4\sqrt{3}, -5\sqrt{3}$$

$$-10, -12, -14$$

Ex1: Find the 41st term in the arithmetic sequence $\boxed{-12}$, -1 , 10 , \dots

$+11$ $+11$ common difference

$$\text{term } 41 = -12 + 40(11)$$

$$= -12 + 440$$

$$= \boxed{428}$$

Ex1: Find the 41st term in the arithmetic sequence $-12, -1, 10, \dots$

$$* a_n = a_1 + (n-1)d$$

a = term

a_n : n^{th} or last term

$$a_{41} = -12 + (41-1)11$$

a_1 = first term

$$= -12 + 40(11)$$

n = number of terms

$$= 428$$

d = common difference

Ex 2: Find the in the arithmetic sequence for which $a_{44} = 229$ and $d = 8$.

$$a_1 = 229 + 43(-8)$$

$$a_1 = 229 - 344$$

$$a_1 = -115$$

$$a_n = a_1 + (n-1)d$$

$$229 = a_1 + (44-1)(8)$$

$$229 = a_1 + (+344) - 344$$

$$-115 = a_1$$

EX3 Write an arithmetic sequence that has arithmetic means -12 and 23 .

$-12, \wedge -1, \wedge -2, \wedge 3, \wedge 8, \wedge 13, \wedge 18, \wedge 23$

$$\frac{35}{7} = 5$$

$$a_n = a_1 + (n-1)d$$

$$23 = -12 + (8-1)d$$

$$\frac{35}{7} = \frac{7d}{7}$$

$$d = 5$$

Karl Friedrich Gauss

Ex4: Find the sum of all the numbers 1 to 100.

$$1 + 2 + 3 + \dots + 96 + 97 + 98 + 99 + 100$$

$$\begin{array}{r} \underline{\underline{50}} (100) = 5000 \\ \underline{\quad 50} \\ 101 \end{array}$$

$$\boxed{S_n = \frac{n}{2} (a_1 + a_n)} \quad \begin{array}{r} 5050 \\ \underline{\quad 50} \\ 101 \end{array} = 50(101) = 5050$$

$$\begin{aligned} S_{100} &= \frac{100}{2} (1 + 100) \\ &= 50 (101) = 5050 \end{aligned}$$

Never ASSUME!

EXS: Find the sum of the first 32 terms in the arithmetic series $-12 - 6 - 0 + \dots + \boxed{174}$.
 $+6 +6$

$$a_{32} = -12 + (31)6$$
$$= -12 + 186$$
$$= 174$$

$$S_{32} = 16(174 + (-12))$$
$$= 16(162)$$

$$S = 2592$$

$$\begin{array}{r} 3162 \\ \underline{16} \\ 972 \\ \underline{162} \\ 2592 \end{array}$$

$$S_{32} = \frac{32}{2}(-12 + 174)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

"Sum"

Ex 4:

Evaluate:

$$\sum_{k=3}^{10}$$

$$(2k+1)$$

$$= 7 + 9 + 11 + 13 + 15$$

$$+ 17 + 19 + 21$$

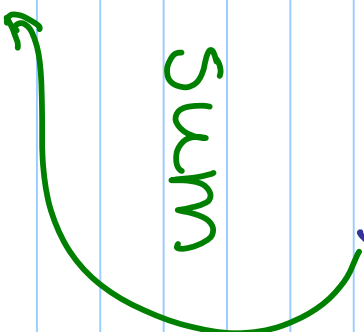
Expand

$$\sum$$

sigma

=

sum



$$= \frac{8}{2} (7 + 21)$$

$$= 4(28)$$

$$= 112$$

HW p 763 (17-47) odd