Solving Exponential Equations

Note Title 9/29/2010

I can expand and simplify logarithms using Log Laws, and I can solve an exponential equation.

common logs : $\log_{10}$ written $\boxed{\log}$

natural logs : $\log_e$ written $\boxed{\ln}$

$e \approx 2.71$

exponential form	logarithic form
$b^p = n$	$\log_b n = p$
$2^3 = 8$	$\log_2 8 = 3$
$9^{\frac{1}{2}} = 3$	$\log_9 3 = \frac{1}{2}$

23) $20 \log_6 u^{20} + 5 \log_6 v^5 = \log_6 u^{20} v^5$

$\frac{1}{2} \log x^{\frac{1}{2}} - \frac{2}{3} \log y^{3/3} + 5 \log z^5 - \log 3$

$\log \frac{x^{1/2} z^5}{3y^{2/3}}$

Solve.

① $3^{2x} = 27^{x+1}$

- Like bases!
- Set exponent = & solve

$3^{2x} = (3^3)^{x+1}$

$3^{2x} = 3^{3x+3}$

$2x = 3x + 3$

$-3 \quad -2x \qquad -2x \quad -3$

$-3 = x$

Log Laws

$$x^a \cdot x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

① $\log_b x + \log_b = y = \log_b xy$

② $\log_b x - \log_b = \log_b \frac{x}{y}$

③ $\log_b x^a = a \log_b x$

- $\log \frac{\sqrt{x y^2}}{5}$
- $\log \frac{(x y^2)^{\frac{1}{3}}}{5}$

④ If $\log_b x = \log_b y$, then $x = y$.

- $\log \frac{x^{\frac{1}{2}} y}{5}$

⑦ $\log \frac{x}{y^6} = \log x - 6 \log y$

$\frac{1}{2} \log x + \log y - \log 5$

17) $64 \cdot 16^{-3x} = 16^{3x-2}$

$4^3 \cdot (4^2)^{-3x} = (4^2)^{3x-2}$

$4^3 \cdot 4^{-6x} = 4^{6x-4}$

$4^{3-6x} = 4^{6x-4}$

$3 - 6x = 6x - 4$

$4 + 6x = +6x + 4$

$7 = 12x$

$\frac{7}{12} = x$

- Like bases!
- Set exponents = and solve!

Ex1 : Solve. $\log 5^{4x} = \log 73$

$$\frac{4x \log 5}{4 \log 5} = \frac{\log 73}{4 \log 5}$$

* calculator ready $X = \dfrac{\log(73)}{(4\log(5))}$ always ( ) !

$$X = 0.666$$

Ex 2: $\log 2^{x-1} = \log 5^{x-2}$

① log BOTH sides;

$(x-1)\log 2 = (x-2)\log 5$

② (LL #3 to bring down exponents.)

$x\log 2 - \log 2 = x\log 5 - 2\log 5$

③ Distribute

$-x\log 5 + \log 2 \qquad -x\log 5 + \log 2$

$x\log 2 - x\log 5 = -2\log 5 + \log 2$

④ x's together

$x(\log 2 - \log 5) = -2\log 5 + \log 2$

⑤ factor out x

$(\log 2 - \log 5) \qquad (\log 2 - \log 5)$

⑥ Divide

$$x = \frac{-2\log(5) + \log(2)}{(\log(2) - \log(5))} = \boxed{2.756}$$

⑦ Calculator = before ÷

Ex3: Solve $10 = 2e^{3k}$

b) $25 > e^{0.2t}$