

By Definition a function has symmetry about the **y-axis if $f(-x) = f(x)$** . Which means that if you put a $-x$ in for the x 's of the function and you end up with the same function you started with, then the graph has y axis symmetry.

Example: $y = \frac{x^2 - 4}{x^4 - x^2}$ $f(-x) = \frac{(-x)^2 - 4}{(-x)^4 - (-x)^2} = \frac{x^2 - 4}{x^4 - x^2} = f(x)$ so this function has y-axis symmetry.

A function with **y-axis** symmetry is called an **even** function.

There is a shortcut to see if polynomials have y-axis symmetry. If all of the powers of x are even then the function is even; therefore, it has y-axis symmetry.

Example $f(x) = x^4 + x^2 - 1$ is even. (The x of the constant term is x^0 and 0 is considered even)

By Definition a function has symmetry about the **origin if $f(-x) = -f(x)$** . Which means that if you put a $-x$ in for the x 's of the function and you end up with the negative of function you started with, then the graph has origin symmetry.

Example: $y = \frac{x^3 - 4x}{x^4 - x^2}$ $f(-x) = \frac{(-x)^3 - 4(-x)}{(-x)^4 - (-x)^2} = \frac{-x^3 + 4}{x^4 - x^2} = \frac{-(x^3 - 4)}{x^4 - x^2} = -f(x)$ so this function has origin

symmetry.

A function with **origin** symmetry is called an **odd** function.

There is a shortcut to see if polynomials have origin symmetry. If all of the powers of x are odd then the function is odd; therefore, it has origin symmetry.

Example $f(x) = x^3 + x$ is odd.

If you mix up the types of powers then we say that the function has no symmetry.

Example $f(x) = x^3 - 5$ has not symmetry (Remember constants are considered even because they have an x^0 there)

If you have a rational function, then you treat the functions like signed numbers. Determine the symmetry of the numerator and denominator.

A) If it is even (y axis symmetry) treat it like a (+)

B) If it is odd (origin symmetry) treat it like a (-)

$$y = \frac{\text{even}}{\text{odd}} = \frac{+}{-} = - \quad \text{which means that the rational function is odd (has origin symmetry)}$$

$$y = \frac{\text{even}}{\text{even}} = \frac{+}{+} = + \quad \text{which means the rational function is even (has y-axis symmetry)}$$

$$y = \frac{\text{odd}}{\text{odd}} = \frac{-}{-} = + \quad \text{which means the rational function is even (has y-axis symmetry)}$$

If the numerator or denominator has no symmetry, that automatically means that the rational function has no symmetry (y-axis or origin).

I. List the symmetry of the following functions.

_____ 1. $y = \frac{x^3 - x}{x^2 - 2}$

_____ 6. $y = x^3 - x$

_____ 2. $y = \frac{x}{x^2 - 4}$

_____ 7. $y = x^2 + 3$

_____ 3. $y = \frac{2}{x^2}$

_____ 8. $y = e^x + 2$

_____ 4. $y = \frac{x^2 - x}{x^3 + x - 5}$

_____ 9. $y = \cos x + x^2$

_____ 5. $y = \frac{x^3}{x^5 - x}$

_____ 10. $y = \frac{\sin x}{x^3}$

List the type of function given in each graph. (Polynomial, Rational, Trigonometric, Exponential, Logarithmic, Basic Function that doesn't fit a category)



