

## THINGS TO KNOW ABOUT FUNCTIONS

1. Function- a relationship between two variables such that each value of the independent variable corresponds with exactly one value of the dependent variable.

A quick way to think of this is that each domain value has one and only one range value. (each  $x$  has one  $y$ )

Graphically, a graph is a function if you can draw a vertical line and pass that line across the graph and it only touches the graph at one point (vertical line test).

$f(x)$  is function notation and is said as “ $f$  of  $x$ ” and means a function with an independent variable of  $x$ . Many times  $f(x)$  is used instead of  $y$ .  $G(r)$  is read as “ $g$  of  $r$ ” and that would mean that  $r$  would be the independent variable and  $G(r)$  would be the dependent variable. The value of  $G(r)$  is dependent on the value of  $r$ .

2. Domain- The set of all values of the independent variable for which the function is defined. In a normal  $(x,y)$  function, it is the set of all  $x$ 's for which the function is defined.

3. Range- The set of all values assumed by the dependent variable. In a normal  $(x,y)$  function, it is the set of all  $y$ 's for which the function is defined.

4. In order to discuss a set of values, the use of interval notation is needed. Interval notation uses parenthesis and brackets around two numbers separated by a comma to indicate all of the numbers between the two given numbers. A bracket indicates that the number is included in the interval and a parenthesis indicates that the number is not included.

EX 1  $[2,5)$  indicates all of the numbers between 2 and 5 and includes 2. You say  $[2,5)$  as “the interval from 2 inclusive to 5 not inclusive”

$2 \leq x < 5$  is a notation that you may have seen in earlier math classes that also means the same thing as  $[2,5)$ . Notice that with the interval notation a variable is not needed.

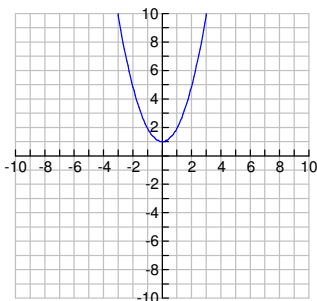
EX 2  $(-1,4)$  indicates all of the numbers between  $-1$  and 4, but does not include  $-1$  or 4. You say  $(-1,4)$  as “the interval from  $-1$  not inclusive to 4 not inclusive”.

$-1 < x < 4$  is a notation that you may have seen in earlier math classes that also means the same thing as  $(-1,4)$ .

EX 3 If your interval starts at one point and goes forever, let's say it starts at 1 and goes infinitely in the positive direction, then the interval notation is  $[1, +\infty)$ . The parenthesis is always used around an infinity symbol because you can never include infinity.

$x \geq 1$  is a notation that you may have seen in earlier math classes that also means the same thing as  $[1, +\infty)$ .

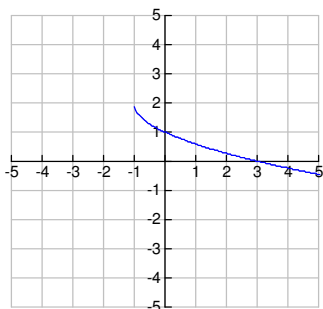
EX 4  $y = x^2 + 1$



The domain is  $(-\infty, +\infty)$ . This means that you can put in any value for  $x$  and you get a real answer. Graphically, the graph goes on forever in the right and left direction without interruption.

The range is  $[1, +\infty)$ . This means that the  $y$ -values start at 1 inclusive (it actually hits  $y=1$ ) and goes up forever.

EX 5  $y = -\sqrt{x+1} + 2$



The domain is found by scanning the graph from left to right.

The domain is  $[-1, +\infty)$ . This means that horizontally the graph starts at  $-1$  (inclusive) and goes right forever.

The range is found by scanning a graph from bottom to top.

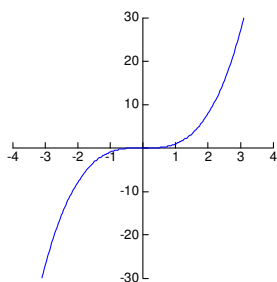
The range is  $(-\infty, 2]$ . This means that if you look at the graph vertically, there really is no bottom value because the graph would go down forever (hence the  $\div$  symbol) and then the  $y$ -values would increase until  $2$ . Because  $2$  is an actual range value, then it is inclusive.

5. Symmetry- There are four different types of symmetry: 1)  $y$ -axis, 2) origin, 3)  $x$ -axis, and 4) point. For our discussion we will be discussing the first two because  $x$ -axis symmetry happens when you are not working with a function and point symmetry is  $y$ -axis and origin symmetry of a translated function.

A)  **$y$ -axis symmetry**- A function has  $y$ -axis symmetry if  $f(-x) = f(x)$ . This means that if you place a number or the negative of that same number into the  $x$  of the function you get the same answer. For example, take the function of  $y = x^2 + 1$  above. If you substitute a  $2$  or  $-2$  into the  $x$ , you get the same answer of  $5$ . If that works for all numbers, which it does, then the function has  $y$ -axis symmetry. The graphical way to determine if a graph has  $y$ -axis symmetry is to fold the graph on the  $y$ -axis. If the right side is folded over the  $y$ -axis and it lands directly on the left side of the graph then the graph has  $y$ -axis symmetry.

If a function has  **$y$ -axis symmetry** then we say that the function is **even**.

B) **origin symmetry**- A function has origin symmetry if  $f(-x) = -f(x)$ . This means that if you place a number into a function you get an answer and if you put the negative of that number into the function you get the negative of the value obtained with the positive number. For example, take the function  $y = x^3$ . Substitute in  $3$  for the  $x$  and you get the value of  $27$ . Substitute the value of  $-3$  and you get  $-27$ .



Graphically, there are a few ways you can determine if a graph has origin symmetry. If from the origin you go right and up to get to a point on the graph, then you should be able to go left and down the same amount and land on the graph. A quick way to determine origin symmetry is to take the right side of the graph and flip it over the  $y$ -axis and the  $x$ -axis. If it lands on the left side of the graph then the graph has origin symmetry. If you hear me say flip-flip, that is what I am talking about.

If a graph has **origin symmetry** then we say that the function is **odd**.

6. **Roots** of a function- The  $x$ -value where the graph crosses the  $x$ -axis is called the root of the function. This crossing place is also referred to as the  **$x$ -intercept**, **zero of the function**, and the **solution**. If a graph hits the  $x$ -axis but does not go through the axis and bounces back, this is called a double root. An example of a double root is  $y = x^2$  where the double root is at  $x=0$ .
7. You normally need a few extra points to help “guide” you when graphing a function. If you will use  $x = 1$  and  $-1$ , that usually is enough.