## TRIGONOMETRY

**Trigonometry** (from Greek *trigōnon*, "triangle" and *metron*, "measure") is a branch of mathematics that studies relationships involving lengths and angles of triangles. The field emerged during the 3rd century BC from applications of geometry to astronomical studies.

The 3rd-century astronomers first noted that the lengths of the sides of a rightangle triangle and the angles between those sides have fixed relationships: that is, if at least the length of one side and the value of one angle are known, then all other angles and lengths can be determined algorithmically. These calculations soon came to be defined as the trigonometric functions and today are pervasive in both pure and applied mathematics

Sumerian astronomers studied angle measure, using a division of circles into 360 degrees.<sup>[4]</sup> They, and later the Babylonians, studied the ratios of the sides of similar triangles and discovered some properties of these ratios but did not turn that into a systematic method for finding sides and angles of triangles. The ancient Nubians used a similar method.

In the 3rd century BCE, classical Greek mathematicians (such as Euclid and Archimedes) studied the properties of chords and inscribed angles in circles, and they proved theorems that are equivalent to modern trigonometric formulae, although they presented them geometrically rather than algebraically.

The modern sine function was first defined in the *Surya Siddhanta*, and its properties were further documented by the 5th century (*CE*) Indian mathematician and astronomer Aryabhata. These Greek and Indian works were translated and expanded by medieval Islamic mathematicians. By the 10th century, Islamic mathematicians were using all six trigonometric functions, had tabulated their values, and were applying them to problems in spherical geometry. At about the same time, Chinese mathematicians developed trigonometry independently, although it was not a major field of study for them. Knowledge of trigonometric functions and methods reached Europe via Latin translations of the works of Persian and Arabic astronometry by a European mathematician is *De Triangulis* by the 15th century German mathematician Regiomontanus. Trigonometry was still so little known in 16th-century Europe that Nicolaus Copernicus devoted two chapters of *De revolutionibus orbium coelestium* to explain its basic concepts. Driven by the demands of navigation and the growing need for accurate maps of large geographic areas, trigonometry grew into a major branch of mathematics. Bartholomaeus Pitiscus was the first to use the word, publishing his *Trigonometria* in 1595. Gemma Frisius described for the first time the method of triangulation still used today in surveying. It was Leonhard Euler who fully incorporated complex numbers into trigonometry. The works of James Gregory in the 17th century and Colin Maclaurin in the 18th century were influential in the development of trigonometric series. Also in the 18th century, Brook Taylor defined the general Taylor series. There are many uses of trigonometry and trigonometric functions. For instance, the technique of triangulation is used in astronomy to measure the distance to nearby stars, in geography to measure distances between landmarks, and in satellite navigation systems. The sine and cosine functions are fundamental to the theory of periodic functions such as those that describe sound and light waves.

Fields that use trigonometry or trigonometric functions include astronomy (especially for locating apparent positions of celestial objects, in which spherical trigonometry is essential) and hence navigation (on the oceans, in aircraft, and in space), music theory, audio synthesis, acoustics, optics, electronics, probability theory, statistics, biology, medical imaging (CAT scans and ultrasound), pharmacy, chemistry, number theory (and hence cryptology), seismology, meteorology, oceanography, many physical sciences, land surveying and geodesy, architecture, image compression, phonetics, economics, electrical engineering, mechanical engineering, civil engineering, computer graphics, cartography, crystallography and game development.