

Exercises

a

Prove each identity.

1. $\tan \theta = \sin \theta \cdot \sec \theta$

3. $\tan^2 \theta = \frac{1 - \cos^2 \theta}{\cos^2 \theta}$

5. $\tan^2 \theta = \sec^2 \theta - 1$

7. $\frac{\cos^2 \theta}{\sin \theta} + \sin \theta = \csc \theta$

9. $\csc \theta = \frac{\cot \theta}{\cos \theta}$

10. $\frac{1}{\sec^2 \theta} + \frac{1}{\csc^2 \theta} = 1$

11. $\csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$

12. $\frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} = 1$

13. $\csc^4 \theta - \cot^4 \theta = \csc^2 \theta + \cot^2 \theta$

14. $\sec^4 \theta - \tan^4 \theta = \tan^2 \theta + \sec^2 \theta$

15. $(1 - \tan \theta)^2 = \sec^2 \theta - 2 \tan \theta$

16. $(1 - \sin^2 \theta)(1 + \tan^2 \theta) = 1$

17. $\frac{\cot \theta}{\cos \theta} + \frac{\sec \theta}{\cot \theta} = \sec^2 \theta \cdot \csc \theta$

18. $2 \sin^2 \theta - 1 = 1 - 2 \cos^2 \theta$

19. $\sec \theta - \tan \theta \cdot \sin \theta = \cos \theta$

b

20. $\frac{\sec \theta \cdot \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$

21. $\csc \theta + \cot \theta = \frac{1 + \cos \theta}{\sin \theta}$

22. $\cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta$

23. $\cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$

24. $\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$

25. $\tan^2 \theta \sin^2 \theta = \tan^2 \theta - \sin^2 \theta$

26. $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

27. $(\sin \theta + \cos \theta)(\tan \theta + \cot \theta) = \sec \theta + \csc \theta$

2. $\cot \theta = \cos \theta \cdot \csc \theta$

4. $\sec^2 \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}$

6. $\cot^2 \theta = \csc^2 \theta - 1$

8. $\frac{\tan \theta}{1 - \cos^2 \theta} = \sec \theta \cdot \csc \theta$

28. $\frac{\tan \theta - 1}{\tan \theta + 1} = \frac{1 - \cot \theta}{1 + \cot \theta}$

29. $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = 1 - 2 \sin^2 \theta$

30. $\frac{\cos \theta + 1}{\sin^2 \theta} = \frac{\csc \theta}{1 - \cos \theta}$

31. $\frac{\sin \theta}{1 - \cos \theta} = \csc \theta + \cot \theta$

32. $\frac{\tan \theta}{\sec \theta} + \frac{\cot \theta}{\csc \theta} = \sin \theta + \cos \theta$

33. $\frac{\sin \theta + \tan \theta}{1 + \sec \theta} = \sin \theta$

34. $\frac{2 \tan \theta}{1 + \tan^2 \theta} = 2 \sin \theta \cos \theta$

35. $\frac{1 + \cot \theta}{\csc \theta} = \frac{1 + \tan \theta}{\sec \theta}$

36. $\cos^4 \theta - \sin^4 \theta = 2 \cos^2 \theta - 1$

37. $\frac{1 - 2 \sin \theta - 3 \sin^2 \theta}{\cos^2 \theta} = \frac{1 - 3 \sin \theta}{1 - \sin \theta}$

38. $(\tan x + \cot x)^2 = \sec^4 x \cot^2 x$

39. $(\sec x + \tan x) = \frac{\cos x}{1 - \sin x}$

40. $\cos^4 x - \sin^4 x - 1 = -2 \sin^2 x$