

**Calculus 1 Worksheet #21**  
**Derivatives of Inverse Trig Functions and Implicit Differentiation**

**EXAMPLES:**

1. The derivative of  $y = \cos^{-1}(5x)$  is  $-\frac{1}{\sqrt{1-u^2}}(d\Box) = -\frac{1}{\sqrt{1-(5x)^2}}(5) = \boxed{\boxed{-\frac{5}{\sqrt{1-25x^2}}}}$

2. The derivative of  $y = \tan^{-1}(4x^3)$  is  $\frac{1}{1+u^2}(d\Box) = \frac{1}{1+(4x^3)^2}(12x^2) = \boxed{\boxed{\frac{12x^2}{1+16x^6}}}$

**EXAMPLE FOR IMPLICIT DIFFERENTIATION :**

1. Find  $dy/dx$  for  $y^2 - 4x^3 + xy = 23$

3.  $2y\left(\frac{dy}{dx}\right) - 12x^2\left(\frac{dx}{dx}\right) + \left[\left(\frac{dx}{dx}\right)y + x\left(\frac{dy}{dx}\right)\right] = 0$

$2y\left(\frac{dy}{dx}\right) - 12x^2 + y + x\left(\frac{dy}{dx}\right) = 0 \Rightarrow 2y\left(\frac{dy}{dx}\right) + x\left(\frac{dy}{dx}\right) = 12x^2 - y \Rightarrow$

$(2y + x)\left(\frac{dy}{dx}\right) = 12x^2 - y \Rightarrow \frac{dy}{dx} = \frac{12x^2 - y}{2y + x}$

For the following fourteen problems, find  $\frac{dy}{dx}$ .

1.  $y = \sin^{-1}(2x)$

2.  $y = \tan^{-1}(3x)$

3.  $y = \sec^{-1}(e^{2x})$

4.  $y = \sin^{-1}\sqrt{x}$

5.  $y = \sin^{-1}\left(\frac{x}{3}\right)$

6.  $y = \cos^{-1}(2x + 1)$

7.  $y = \sec^{-1}(x^7)$

8.  $y = \csc^{-1}(e^x)$

9.  $y = \sin^{-1}\left(\frac{1}{x}\right)$

10.  $y = e^x(\sec^{-1} x)$

11.  $y = x^2(\sin^{-1} x)^3$

12.  $y^2 - 8y + x^2 = 5$

13.  $x^2y^3 = 25$

14.  $y^3 - 6y + x^2 = 0$

**Answers:**

1. $\frac{2}{\sqrt{1-4x^2}}$	2. $\frac{3}{1+9x^2}$	3. $\frac{2}{\sqrt{e^{4x}-1}}$	4. $\frac{1}{2\sqrt{x-x^2}}$
5. $\frac{1}{\sqrt{9-x^2}}$	6. $\frac{-2}{\sqrt{1-(2x+1)^2}}$	7. $\frac{7}{ x \sqrt{x^{14}-1}}$	8. $\frac{-1}{\sqrt{e^{2x}-1}}$
9. $\frac{-1}{ x \sqrt{x^2-1}}$	10. $\frac{e^x}{ x \sqrt{x^2-1}} + e^x \sec^{-1} x$	11. $\frac{3x^2(\sin^{-1} x)^2}{\sqrt{1-x^2}} + 2x(\sin^{-1} x)^3$	12. $\frac{-x}{y-4}$
13. $-\frac{2y}{3x}$	14. $-\frac{2x}{3(y^2-2)}$		