

Name _____

Differential Equations WS

Do not use your calculator to solve the differential equation.

Solve the initial value problem by separation of variables.

1. $\frac{dy}{dx} = \frac{x^2}{y}$ given $y = -5$ when $x = 3$

2. $\frac{dy}{dx} = 6x^2y$ and $y(0) = 4$

3. $\frac{dy}{dx} = \frac{1}{y^2}$ and $y(0) = 4$

4. $\frac{dy}{dx} = \frac{1+x}{\sqrt{y}}$ and $y(2) = 9$

5. $\frac{dy}{dx} = -xy^2$ and $y(1) = -0.25$

6. $\frac{dy}{dx} = \frac{4\sqrt{y} \ln x}{x}$ and $y(e) = 9$

7. Find an equation of the curve that satisfies $\frac{dy}{dx} = 4x^3y$ and whose y -intercept is 7.

8. Find an equation of the curve that passes through the point $(1, 1)$ and whose slope at (x, y) is $\frac{y^2}{x^3}$.

9. If $\frac{dy}{dt} = -3y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{3}$?

10. If $\frac{dy}{dx} = y \cos x$ and $y = 3$ when $x = 0$, then $y = ?$

11. If $f'(x) = 2f(x)$ and $f(2) = 1$, then $f(x) = ?$

12. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

- (A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

13. If $\frac{dy}{dx} = x^2y$, then y could be

- (A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

14. If the graph of $y = f(x)$ contains the point $(0, -5)$ and if $\frac{dy}{dx} = \frac{x-3}{y}$, then $f(x) =$

(A) $\sqrt{x^2 - 6x}$

(B) $\sqrt{x^2 - 6x + 25}$

(C) $-\sqrt{x^2 - 6x + 25}$

(D) $\frac{x^2}{2} - 3x + 5$

(E) $\frac{(x-3)^2}{2} + 5$

15. Suppose $\frac{df}{dx} = 4x^{1/2}$. If $f(4) = 21$, then $f(x) =$

(A) $\sqrt{x^3} + 2$

(B) $\frac{8}{3}x^{3/2} - \frac{1}{3}$

(C) $-\frac{3}{8}(1-2x^2)^{2/3}$

(D) $\frac{1}{3}(x+4)^3$

(E) $6x^{3/2} - 27$

16. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1

(B) $-\frac{1}{3}$

(C) 0

(D) $\frac{1}{3}$

(E) 1

17. (no calc) The function f is differential for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6-2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6-2x)$ with the initial condition $f(3) = \frac{1}{4}$.

18. (no calc) Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).